期中复习

2022年4月21日 星期四　下午6:35

空间解析几何.　旋转.柱.锥.相关题.

平面
- 一般方程（点法决定）
  - $P_0(x_0,y_0,z_0)$，$\vec{n}=(a,b,c)$.
  - P满足 $a(x-x_0)+b(y-y_0)+c(z-z_0)=0$
- 三点决定
  - $P_1(x_1,y_1,z_1)$. $P_2(x_2,y_2,z_2)$. $P_3(x_3,y_3,z_3)$.
  - P满足 $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_1-x_2 & y_1-y_2 & z_1-z_2 \\ x_2-x_3 & y_2-y_3 & z_2-z_3 \end{vmatrix}=0$.
- 平面关系 → 法向量关系
- 点到平面的距离
  - $P(x,y,z)$，$\Pi: ax+by+cz+d=0$.
  - $l=\dfrac{|ax+by+cz+d|}{\sqrt{a^2+b^2+c^2}}=\left|\dfrac{\vec{n}\cdot\overrightarrow{PP_0}}{|\vec{n}|}\right|$
  - （P_0为平面上任一点）

直线
- 点向式方程
  - $P_0(x_0,y_0,z_0)$. $\vec{v}=(l,m,n)$.
  - $\dfrac{x-x_0}{l}=\dfrac{y-y_0}{m}=\dfrac{z-z_0}{n}$
  - 或 $x=lt+x_0$, $y=mt+y_0$, $z=nt+z_0$
- 两平面交线.
  - $\begin{cases} a_1x+b_1y+c_1z+d_1=0 \\ a_2x+b_2y+c_2z+d_2=0 \end{cases}$.
  - 取满足的交点 $P(x_0,y_0,z_0)$,
  - 并 $\vec{v}=\vec{n_1}\times\vec{n_2}$，转为一般式.
- 两直线位置关系
  - $\begin{cases} L_1: \vec{r}=\vec{r_1}+t\vec{v_1} \\ L_2: \vec{r}=\vec{r_2}+t\vec{v_2} \end{cases}$
  - 夹角 $\cos\theta=\dfrac{\vec{v_1}\cdot\vec{v_2}}{|\vec{v_1}||\vec{v_2}|}$
  - 共面 $(\vec{r_1}-\vec{r_2})\cdot\vec{v_1}\times\vec{v_2}=0$.
- 点到直线的距离.
  - $P(x,y,z)$. $L: \vec{r}=\vec{r_0}+t\vec{v}$
  - $l=\dfrac{|\vec{v}\times\overrightarrow{P_0P}|}{|\vec{v}|}$　$\vec{r_0}=\overrightarrow{OP_0}=(x_0,y_0,z_0)$
- 直线与平面位置
  - 夹角 $\sin\theta=|\cos\theta|=\dfrac{|\vec{n}\cdot\vec{v}|}{|\vec{n}||\vec{v}|}$

曲面
- 柱面：空间曲线 $L: \begin{cases} F(x,y)=0 \\ z=0 \end{cases}$ 作准线.
  - 方向向量 $\vec{v}$ 作母线
  - $\dfrac{x-x_1}{l}=\dfrac{y-y_1}{m}=\dfrac{z-z_1}{n}$
- 锥面：空间曲线 L 作准线
  - 不在 L 上的点 O 与 L 上的点
  - 连线作母线.
  - $\dfrac{x-x_1}{x_1-x_0}=\dfrac{y-y_1}{y_1-y_0}=\dfrac{z-z_1}{z_1-z_0}$
- 旋转曲面
- 椭球面. 双曲面.
- 二次锥面. 抛物面.

空间曲线/面.
- 曲线
  - 参
    - 表示：$\vec{r}(t)=x(t)\vec{i}+y(t)\vec{j}+z(t)\vec{k}$
    - 切线：$\dfrac{x-x(t_0)}{x'(t_0)}=\dfrac{y-y(t_0)}{y'(t_0)}=\dfrac{z-z(t_0)}{z'(t_0)}$
    - 法平面：$x'(t_0)(x-x(t_0))+y'(t_0)\cdots+z'(t_0)\cdots=0$.
    - 弧长：$l=\int_\alpha^\beta|\vec{r}'(t)|\,dt=\int_\alpha^\beta\sqrt{x'(t)^2+y'(t)^2+z'(t)^2}\,dt$.
    - 曲率：$\vec{k}=\vec{r}'\times\vec{r}''=\dfrac{\vec{r}'(t)\times\vec{r}''(t)}{|\vec{r}'(t)|^3}$.
  - 隐
    - 表示：$\begin{cases} F(x,y,z)=0 \\ G(x,y,z)=0 \end{cases}$
    - 切线 $\left(\dfrac{\partial(F,G)}{\partial(y,z)},\dfrac{\partial(F,G)}{\partial(z,x)},\dfrac{\partial(F,G)}{\partial(x,y)}\right)$
- 曲面
  - 参
    - 表示：$\vec{r}=\vec{r}(u,v)=(x(u,v),y(u,v),z(u,v))$
    - $\vec{r}_u'=\left(\dfrac{\partial x}{\partial u},\dfrac{\partial y}{\partial u},\dfrac{\partial z}{\partial u}\right)$，$\vec{r}_v'=\left(\dfrac{\partial x}{\partial v},\dfrac{\partial y}{\partial v},\dfrac{\partial z}{\partial v}\right)$
    - $\vec{r}_u'\times\vec{r}_v'=\left(\dfrac{\partial(y,z)}{\partial(u,v)},\dfrac{\partial(z,x)}{\partial(u,v)},\dfrac{\partial(x,y)}{\partial(u,v)}\right)$
    - $\Rightarrow r_x'=(1,0,f_x')$，$r_y'=(0,1,f_y')$
    - $\vec{n}=\vec{r}_x'\times\vec{r}_y'=(-f_x',-f_y',1)$.
    - 切平面 $z-f(x_0,y_0)=f_x'(x-x_0)+f_y'(y-y_0)$
    - 法线 $\dfrac{x-x_0}{f_x'}=\dfrac{y-y_0}{f_y'}=\dfrac{z-f(x_0,y_0)}{-1}$
  - 隐
    - 表示：$F(x,y,z)=0$.
    - 法向量 $\vec{n}=\operatorname{grad}F=(F_x',F_y',F_z')$
    - 切平面 $F_x'(x-x_0)+F_y'(y-y_0)+F_z'(z-z_0)=0$

多元函数的微分.

平面点集基本概念.

极限.
- 定义：设 $z=f(x,y)$，$(x_0,y_0)\in D$.
  - $\forall\varepsilon>0$，$\exists\delta>0$，当 $0<\rho(M,M_0)<\delta$ 时，
  - 有 $|f(x,y)-A|<\varepsilon$，则 $\lim\limits_{(x,y)\to(x_0,y_0)}f(x,y)=A$.
- 定理：点列. 复合.
  - 累次极限：先对一个，再对一个
  - 多元极限：同时，按邻域.
- 极限不存在：可能沿不同路径，不同.

连续.
- 定义：$\lim\limits_{(x,y)\to(x_0,y_0)}f(x,y)=f(x_0,y_0)$
  - 对 $\forall(x_0,y_0)\in D$ 而言：
  - 逐点连续：对 $\forall\varepsilon>0$，$\exists\delta$（随着 (x_0,y_0) 改变），当 $0<\rho(M,M_0)<\delta$，有 $|f(x,y)-f(x_0,y_0)|<\varepsilon$.
  - 一致连续：对 $\forall\varepsilon>0$，$\exists\delta$，对 $\forall(x_0,y_0)\in D$，$\rho(M,M_0)<\delta$，有 $|f(x,y)-f(x_0,y_0)|<\varepsilon$.
  - 不连续：可能沿不同路径.
- 性质：同单变量.

微分
- 偏导数：$\dfrac{\partial f}{\partial x}(x_0,y_0)=\lim\limits_{x\to x_0}\dfrac{f(x,y)-f(x_0,y)}{x-x_0}$
  - （其他变量看作常数）
- 高阶偏导：$\dfrac{\partial^n f}{\partial x^i\partial y^{n-i}}$.
  - 若 $\dfrac{\partial^2 f}{\partial x\partial y}$. $\dfrac{\partial^2 f}{\partial y\partial x}$ 均连续，则其相等.
  - 记 $\varphi(x)=f(x,y_0+\Delta y)-f(x,y_0)$
  - $\psi(y)=f(x_0+\Delta x,y)-f(x_0,y)$.
  - $\varphi(x_0+\Delta x)-\varphi(x_0)=\psi(y_0+\Delta y)-\psi(y_0)$
  - 微分中值. $\varphi'(x_0+\theta_1\Delta x)\Delta x=\psi'(y_0+\theta_2\Delta y)\Delta y$.
  - $[f_y'(x_0+\Delta x,y_0+\theta_2\Delta y)-f_y'(x_0,y_0+\theta_2\Delta y)]\Delta y$
  - $=[f_x'(x_0+\theta_1\Delta x,y_0+\Delta y)-f_x'(x_0+\theta_1\Delta x,y_0)]\Delta x$
  - $f_{xy}''(x_0+\theta_1\Delta x,y_0+\gamma_1\Delta y)=f_{yx}''(x_0+\gamma_2\Delta x,y_0+\theta_2\Delta y)$.
  - $\therefore f_{xy}''(x_0,y_0)=f_{yx}''(x_0,y_0)$.
- 函数可微
  - 定义：$\Delta z=a\Delta x+b\Delta y+o(\rho)$
    - $\therefore df(x,y)=a\,dx+b\,dy$,
    - 且 $a=\dfrac{\partial f(x,y)}{\partial x}$，$b=\dfrac{\partial f(x,y)}{\partial y}$.
  - 性质
    - 可微 ⇒ 偏导存在（定义）.
    - 可微 ⇒ 连续.
      - 可微，则 $\exists a,b$ 使
      - $f(x+\Delta x,y+\Delta y)-f(x,y)=a\Delta x+b\Delta y+o(\rho)$.
      - $\therefore|f(x+\Delta x,y+\Delta y)-f(x,y)|=|a\Delta x+b\Delta y+o(\rho)|$
      - $\le|a\Delta x|+|b\Delta y|+o(\rho)$.
      - $\le|a|\rho+|b|\rho+o(\rho)$
      - 对 $\forall\varepsilon>0$，取 $\delta=\dfrac{1}{|a|+|b|}\varepsilon$.
      - 则当 $0<\rho<\delta$ 时，
      - $|f(x+\Delta x,y+\Delta y)-f(x,y)|\le\varepsilon+o(\varepsilon)$.
      - $\therefore\lim\limits_{\substack{\Delta x\to0\\\Delta y\to0}}f(x+\Delta x,y+\Delta y)=f(x,y)$.
    - 有偏导 $\xRightarrow{\text{偏导有界}}$ 连续.
      - 有偏导，则 $\lim\limits_{\Delta x\to0}\dfrac{f(x+\Delta x,y)-f(x,y)}{\Delta x}=\dfrac{\partial f(x,y)}{\partial x}$
      - $\lim\limits_{\Delta y\to0}\dfrac{f(x,y+\Delta y)-f(x,y)}{\Delta y}=\dfrac{\partial f(x,y)}{\partial y}$
      - 偏导有界，则 $\exists M$，使对 $\forall(x,y)\in D$，
      - 有 $\left|\dfrac{\partial f(x,y)}{\partial x}\right|<M$，$\left|\dfrac{\partial f(x,y)}{\partial y}\right|<M$
      - （要连续）对 $\forall\varepsilon>0$，$\exists\delta=\dfrac{1}{2M}\varepsilon$，当 $0<\Delta x,\Delta y<\delta$.
      - $|f(x+\Delta x,y+\Delta y)-f(x,y)|$
      - $=|f(x+\Delta x,y+\Delta y)-f(x,y+\Delta y)+f(x,y+\Delta y)-f(x,y)|$
      - $\le|f(x+\Delta x,y+\Delta y)-f(x,y+\Delta y)|+|f(x,y+\Delta y)-f(x,y)|$
      - $=\left|\dfrac{\partial f}{\partial x}(x,y+\theta_1\Delta y)\Delta y\right|+\left|\dfrac{\partial f}{\partial x}(x+\theta_2\Delta x,y)\Delta x\right|$
      - $<|M\Delta y|+|M\Delta x|<2M\delta=\varepsilon$. 连续.
    - 有偏导 ⟹ 可微.
      - 偏导连续，对 $\forall\varepsilon$，$\exists\delta_0$，当 $0<\Delta x,\Delta y<\varepsilon_0$ 时.
      - 有 $\left|\dfrac{\partial f}{\partial x}(x,y+\Delta y)-\dfrac{\partial f}{\partial x}(x,y)\right|<\dfrac{\varepsilon}{2}$，$\left|\dfrac{\partial f}{\partial y}(x+\Delta x,y)-\dfrac{\partial f}{\partial y}(x,y)\right|<\dfrac{\varepsilon}{2}$
      - （要可微，$\lim\limits_{\rho\to0}\dfrac{f(x+\Delta x,y+\Delta y)-f(x,y)-a\Delta x-b\Delta y}{\rho}=0$
      - 其中 $a=\dfrac{\partial f(x,y)}{\partial x}$，$b=\dfrac{\partial f(x,y)}{\partial y}$.）
      - 对 $\forall\varepsilon>0$，存在 $\delta=\delta_0$，当 $0<\Delta x,\Delta y<\varepsilon$ 时，
      - $\left|\dfrac{f(x+\Delta x,y+\Delta y)-f(x,y)-\frac{\partial f(x,y)}{\partial x}\Delta x-\frac{\partial f(x,y)}{\partial y}\Delta y}{\rho}\right|$
      - $\le\left|\dfrac{\left[\frac{\partial f}{\partial x}(x,y+\theta_1\Delta y)-\frac{\partial f}{\partial x}(x,y)\right]\Delta x+\left[\frac{\partial f}{\partial y}(x+\theta_2\Delta x,y)-\frac{\partial f}{\partial y}(x,y)\right]\Delta y}{\rho}\right|$
      - $\le\left|\dfrac{\partial f}{\partial x}(x,y+\theta_1\Delta y)-\dfrac{\partial f}{\partial x}(x,y)\right|+\left|\dfrac{\partial f}{\partial y}(x+\theta_2\Delta x,y)-\dfrac{\partial f}{\partial x}(x,y)\right|$.
      - $<\dfrac{\varepsilon}{2}+\dfrac{\varepsilon}{2}=\varepsilon$. 即 $\lim\limits_{\rho\to0}\dfrac{f(x+\Delta x,y+\Delta y)-f(x,y)-a\Delta x-b\Delta y}{\rho}=0$

方向导数.
- 定义：$M_0(x_0,y_0)$，取方向向量 $\vec{e}=(\cos\alpha,\cos\beta)$.
  - 则直线 L 上的点 $(x,y)=(x_0+t\cos\alpha,y_0+t\cos\beta)$.
  - $\therefore\lim\limits_{t\to0}\dfrac{f(x_0+t\cos\alpha,y_0+t\cos\beta)-f(x_0,y_0)}{t}=\dfrac{\partial f}{\partial\vec{e}}(x_0,y_0)$
- 计算：$\lim\limits_{t\to0}\dfrac{f(x+t\cos\alpha,y+t\cos\beta)-f(x,y)}{t}$
  - $=\lim\limits_{t\to0}\dfrac{f(x+t\cos\alpha,y+t\cos\beta)-f(x,y+t\cos\beta)+f(x,y+t\cos\beta)-f(x,y)}{t}$
  - $=\lim\limits_{t\to0}\dfrac{f_x'(x+\theta_1t\cos\alpha,y+t\cos\beta)t\cos\alpha+f_y'(x,y+\theta_2t\cos\beta)t\cos\beta}{t}$
  - $=\lim\limits_{t\to0}f_x'(x+\theta_1t\cos\alpha,y+t\cos\beta)\cos\alpha+f_y'(x,y+\theta_2t\cos\beta)\cos\beta$
  - $=\dfrac{\partial f}{\partial x}(x,y)\cos\alpha+\dfrac{\partial f}{\partial y}(x,y)\cos\beta$
  - $=\left(\dfrac{\partial f}{\partial x}(x,y),\dfrac{\partial f}{\partial y}(x,y)\right)\cdot(\cos\alpha,\cos\beta)$.
  - $=\operatorname{grad}f\cdot\vec{e}=\nabla f\cdot\vec{e}$.

复合函数
- 定理：设 $u=f(\xi,\eta)$ 在 Ω 中可微，
  - $\xi=\varphi(x,y)$，η 在 D 中可微，
  - 则复合函数 $u=f(\varphi(x,y),\eta(x,y))$ 在 D 可微.
  - 可微：$\Delta u=f(\xi+\Delta\xi,\eta+\Delta\eta)-f(\xi,\eta)$
  - $=a_1\Delta\xi+a_2\Delta\eta+o(\rho)$
  - $\Delta\xi=\varphi(x+\Delta x,y+\Delta y)-\varphi(x,y)$
  - $=b_1\Delta x+b_2\Delta y+o(\tilde\rho)$
  - $\Delta\eta=\psi(x+\Delta x,y+\Delta y)-\psi(x,y)$
  - $=c_1\Delta x+c_2\Delta y+o(\tilde\rho)$
  - $\therefore\Delta u=(a_1b_1+a_2c_1)\Delta x+(a_1b_2+a_2c_2)\Delta y+o(\rho)+o(\tilde\rho)$
  - $\rho=\sqrt{[a_1\Delta x+a_2\Delta y+o(\tilde\rho)]^2+[b_1\Delta x+b_2\Delta y+o(\tilde\rho)]^2}$.
  - $\tilde\rho\to0$ 时，$\rho\to0$，$\dfrac{\rho}{\tilde\rho}$ 有界.
  - $\lim\limits_{\rho\to0}\dfrac{o(\tilde\rho)}{\rho}=\lim\limits_{\rho\to0}\dfrac{o(\tilde\rho)}{\tilde\rho}\cdot\dfrac{\tilde\rho}{\rho}=0$.
  - $\therefore\Delta u=a'\Delta x+b'\Delta y+o(\tilde\rho)$.
- 计算：一阶微分形式的不变性，链式.
  - $\dfrac{\partial u}{\partial x}=\dfrac{\partial u}{\partial\xi}\dfrac{\partial\xi}{\partial x}+\dfrac{\partial u}{\partial\eta}\dfrac{\partial\eta}{\partial x}$.
  - $\dfrac{\partial u}{\partial y}=\dfrac{\partial u}{\partial\xi}\dfrac{\partial\xi}{\partial y}+\dfrac{\partial u}{\partial\eta}\dfrac{\partial\eta}{\partial y}$.
  - 多元→多元，Jacob 行列式.

隐函数
- 求导：对 $F(x,y)$ 若有隐函数 $y=f(x)$，
  - 则 $F(x,f(x))$ 两边对 x 求导，
  - $F_x'+F_y'f'(x)=0$，$f'(x)=-\dfrac{F_x'}{F_y'}$
- 定理：对 $F(x,y)$，$D\subseteq R^2$，
  - 若 ① $F(x,y)\in C^1(D)$ 有连续偏导.
  - ② 存在 $(x_0,y_0)\in D$ 使 $F(x_0,y_0)=0$.
  - ③ 且 $F_y'(x_0,y_0)\neq0$ 或 $F_x'(x_0,y_0)\neq0$.
  - 则 $F(x,y)=0$ 在 (x_0,y_0) 确定隐函数 $y=f(x)$，
  - 且 $\dfrac{dy}{dx}=-\dfrac{F_x'(x,y)}{F_y'(x,y)}$.
- 举例：$F(x,y,z)=0$，$G(x,y,z)=0$，
  - $y=y(x)$，$z=z(x)$.
  - 则 $F_x'+F_y'y'(x)+F_z'z'(x)=0$
  - $G_x'+G_y'y'(x)+G_z'z'(x)=0$.
  - 可以得 $y'(x)$. $z'(x)$.
  - （条件：$\dfrac{\partial(F,G)}{\partial(y,z)}\neq0$.）

逆映射定理.
- 定理：$F(x,y,u,v)=0$
  - $G(x,y,u,v)=0$
  - 决定隐函数 $x=x(u,v)$. $y=y(u,v)$.
  - 若存在逆映射 $u=\varphi(x,y)$，$v=\psi(x,y)$.
  - $\begin{cases} u-\varphi(x,y)=0 \\ v-\psi(x,y)=0. \end{cases}$
  - $\Rightarrow\begin{cases} 1-\varphi_x'x_u'-\varphi_y'y_u'=0 \\ 0-\psi_x'x_u'-\psi_y'y_u'=0. \end{cases}$
  - 得 $\dfrac{\partial x}{\partial u}=\dfrac{1\times\psi_y'-0\times\varphi_y'}{\varphi_x'\psi_y'-\psi_x'\varphi_y'}=\dfrac{\frac{\partial v}{\partial y}}{\frac{\partial u}{\partial x}\frac{\partial v}{\partial y}-\frac{\partial v}{\partial x}\frac{\partial u}{\partial y}}$.
  - $\dfrac{\partial y}{\partial u}=\dfrac{0\times\varphi_x'-1\times\psi_x'}{\varphi_x'\psi_y'-\psi_x'\varphi_y'}=-\dfrac{\frac{\partial v}{\partial x}}{\frac{\partial u}{\partial x}\frac{\partial v}{\partial y}-\frac{\partial v}{\partial x}\frac{\partial u}{\partial y}}$
  - $\therefore\begin{pmatrix}\frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v}\end{pmatrix}=\dfrac{1}{\begin{vmatrix}\frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y}\end{vmatrix}}\begin{pmatrix}\frac{\partial v}{\partial y} & -\frac{\partial u}{\partial y} \\ -\frac{\partial v}{\partial x} & \frac{\partial u}{\partial x}\end{pmatrix}$
  - $=\begin{pmatrix}\frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y}\end{pmatrix}^{-1}$.
- 从微分角度理解.

极值
- 二元函数的 Taylor
  - $f(x,y)=f(x_0+h,y_0+k)$
  - $=\sum\limits_{m=0}^{n}\dfrac{1}{m!}D^mf(x_0,y_0)+\dfrac{1}{(n+1)!}D^{n+1}f(x_0+\theta h,y_0+\theta k)$.
  - 其中算子 $D=h\dfrac{\partial}{\partial x}+k\dfrac{\partial}{\partial y}$.
  - 例 $f^{(2)}(x,y)=h^2\dfrac{\partial^2 f}{\partial x^2}(x_0+th,y_0+tk)+k^2\dfrac{\partial^2 f}{\partial y^2}(x_0+th,y_0+tk)+2hk\dfrac{\partial^2 f}{\partial x\partial y}(x_0+th,y_0+tk)$
  - $f(x,y)=f(x_0,y_0)+\dfrac{\partial f}{\partial x}(x_0,y_0)h+\dfrac{\partial f}{\partial y}(x_0,y_0)k+\dfrac{1}{2}(Ah^2+2Bhk+Ck^2)+R_2$.
  - 余项 $R_n=\dfrac{\partial^{n+1}f}{\partial x^i\partial y^j}(x,y)\dfrac{(n+1)!}{i!\,j!}|h|^i|k|^jM$
- 二元函数微分中值.
  - 设 $f(x,y)$ 在凸区域 D 内有连续偏导，则对 D 内任意两点 (x_1,y_1)，(x_2,y_2)，存在两点连线上一点 (ξ,η) 使得
  - $f(x_2,y_2)-f(x_1,y_1)=(x_2-x_1)f_x'(\xi,\eta)+(y_2-y_1)f_y'(\xi,\eta)$.
- 二元函数极值
  - 内部极值点
    - 驻点 $f_x'(x_0,y_0)=0$，$f_y'(x_0,y_0)=0$.
    - 则 $f(x,y)-f(x_0,y_0)=\dfrac{1}{2}(Ah^2+2Bhk+Ck^2)+o(\rho^2)$.
    - 令 $Q(h,k)=Ah^2+2Bhk+Ck^2$　$\begin{pmatrix}A & B \\ B & C\end{pmatrix}$
    - Q定 ⟺ $AC-B^2>0$，是极值
      - $A>0$，正定，极大.
      - $A<0$，负定，极小.
    - 半定 ⟺ $AC^2-B^2=0$，无法判断.
    - 不定 ⟺ $AC-B^2<0$，不是极值.
  - 边界：条件极值.
    - Lagrange 乘数法. 驻点方程.
    - 有界闭区域内（存在性）

$e^x=\sum\limits_{n=0}^{\infty}\dfrac{x^n}{n!}$

$(1+x)^\alpha=\sum\limits_{n=0}^{\infty}\dfrac{\alpha(\alpha-1)\cdots(\alpha-n+1)}{n!}x^n$.

$\Rightarrow\dfrac{1}{1+x}=\sum\limits_{n=0}^{\infty}(-1)^nx^n$

$\dfrac{1}{\sqrt{1+x}}=\sum\limits_{n=0}^{\infty}\dfrac{(2n-1)!!}{(2n)!!}x^n$.

$\sqrt{1+x}=1+\dfrac{1}{2}x+\sum\limits_{n=2}^{\infty}\dfrac{(2n-3)!!}{(2n)!!}x^n$

$\ln(1+x)=\sum\limits_{n=0}^{\infty}(-1)^n\dfrac{x^{n+1}}{n+1}$.

$\sin x=\sum\limits_{n=0}^{\infty}(-1)^n\dfrac{x^{2n+1}}{(2n+1)!}$.

$\cos x=\sum\limits_{n=0}^{\infty}(-1)^n\dfrac{x^{2n}}{(2n)!}$

$\arctan x=\sum\limits_{n=0}^{\infty}(-1)^n\dfrac{x^{2n+1}}{2n+1}$

多元函数的积分.

二重积分
- 平面区域的面积
- 定义与可积性
- 计算：$\iint\limits_D f(x,y)\,dx\,dy=\int_c^d\int_a^b f(x,y)\,dx\,dy$.　定义+极限
- 换元：$\Delta\sigma=\left|\dfrac{\partial(x,y)}{\partial(u,v)}\right|du\,dv$.　微分中值
  - $=\begin{vmatrix}\frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v}\end{vmatrix}du\,dv$

三重积分.
- 定义
- 计算. $\iiint\limits_V f(x,y,z)\,dx\,dy\,dz$
  - $=\iint\limits_{I_1\times I_2}\left(\int_{I_3}f(x,y,z)\,dz\right)dx\,dy$
  - $=\int_{I_3}\left(\iint\limits_{I_1\times I_2}f(x,y,z)\,dx\,dy\right)dz$
- 换元.

第一型曲线及曲面积分.