知识点重开

2022年2月1日 星期二　　下午10:28

1. 方向导数

方向向量 $\vec{e}=\cos\alpha\,\vec{i}+\cos\beta\,\vec{j}$.

直线上的点 $(x+t\cos\alpha,\ y+t\cos\beta)$.

方向导数 $\dfrac{\partial f}{\partial \vec{e}}=\lim\limits_{t\to 0}\dfrac{f(x+t\cos\alpha,\ y+t\cos\beta)-f(x,y)}{t}$

$=\dfrac{\partial f}{\partial x}\cos\alpha+\dfrac{\partial f}{\partial y}\cos\beta$.

2. 梯度

$\mathrm{grad} f=\dfrac{\partial f}{\partial x}\vec{i}+\dfrac{\partial f}{\partial y}\vec{j}$

$\Rightarrow \dfrac{\partial f}{\partial \vec{e}}=\mathrm{grad} f\cdot\vec{e}=|\mathrm{grad} f|\cos\theta$.

3. 向量值函数.

$\vec{r}(t)=x(t)\vec{i}+y(t)\vec{j}+z(t)\vec{k}$.　$(\mathbb{R}^1\to\mathbb{R}^3)$

$\vec{r}'(t)=x'(t)\vec{i}+y'(t)\vec{j}+z'(t)\vec{k}$

$\vec{f}(\vec{x})=(f_1(x_1,\dots x_n),\ f_2(x_1,\dots x_n),\ \dots,\ f_m(x_1,\dots x_n))^t$.　$(\mathbb{R}^n\to\mathbb{R}^m)$

$f_j'(x_1,\dots x_n)=\sum\limits_{i=1}^{n}\dfrac{\partial f_i}{\partial x_i}$.

$\vec{f}'(\vec{x})=(df_1(\vec{x}),\ df_2(\vec{x}),\ \dots,\ df_m(\vec{x}))$.

$=\begin{pmatrix}\frac{\partial y_1}{\partial x_1} & \cdots & \frac{\partial y_1}{\partial x_n}\\ \vdots & & \vdots\\ \frac{\partial y_m}{\partial x_1} & \cdots & \frac{\partial y_m}{\partial x_n}\end{pmatrix}\begin{pmatrix}dx_1\\ \vdots\\ dx_n\end{pmatrix}$.

4. 隐函数.

$F(x,y)=0$ 决定隐函数 $y=f(x)$, $F(x,f(x))=0$.

求导得 $\dfrac{\partial F}{\partial x}+\dfrac{\partial F}{\partial y}\times\dfrac{\partial y}{\partial x}=0$, $\therefore \dfrac{dy}{dx}=-\dfrac{F_x'}{F_y'}$.

$dF=F_x'dx+F_y'dy$

5. 条件极值 $\to$ Lagrage 乘数法.

6. 向量场的微分.

$\mathbb{R}^3\to\mathbb{R}^3$ 的向量值函数 $\vec{v}(x,y,z)$

$=P(x,y,z)\vec{i}+Q(x,y,z)\vec{j}+R(x,y,z)\vec{k}$.

$\dfrac{\partial\vec{v}}{\partial x}=\dfrac{\partial P}{\partial x}\vec{i}+\dfrac{\partial Q}{\partial x}\vec{j}+\dfrac{\partial R}{\partial x}\vec{k}$.

7. $\nabla=\vec{i}\dfrac{\partial}{\partial x}+\vec{j}\dfrac{\partial}{\partial y}+\vec{k}\dfrac{\partial}{\partial z}$.

梯度 $\nabla\varphi=\dfrac{\partial\varphi}{\partial x}\vec{i}+\dfrac{\partial\varphi}{\partial y}\vec{j}+\dfrac{\partial\varphi}{\partial z}\vec{k}=\mathrm{grad}\,\varphi$.（数乘）

散度 $\nabla\cdot\vec{v}=(\vec{i}\dfrac{\partial}{\partial x}+\vec{j}\dfrac{\partial}{\partial y}+\vec{k}\dfrac{\partial}{\partial z})\cdot(P\vec{i}+Q\vec{j}+R\vec{k})$

$=\dfrac{\partial P}{\partial x}+\dfrac{\partial Q}{\partial y}+\dfrac{\partial R}{\partial z}=\mathrm{div}\,\vec{v}$　点乘).

旋度 $\nabla\times\vec{v}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ \partial_x & \partial_y & \partial_z\\ P & Q & R\end{vmatrix}$　(叉乘).

$=(R_y'-Q_z')\vec{i}+(P_z'-R_x')\vec{j}+(Q_x'+P_y')\vec{k}$

Laplace 算子 $\Delta=\nabla\cdot\nabla$

8. ∇ 在不同坐标下

9. 空间参数式曲线.

$\vec{r}(t)=x(t)\vec{i}+y(t)\vec{j}+z(t)\vec{k}$,

$\vec{r}'(t)=x'(t)\vec{i}+y'(t)\vec{j}+z'(t)\vec{k}$.

切线方程 $\dfrac{x-x(t_0)}{x'(t_0)}=\dfrac{y-y(t_0)}{y'(t_0)}=\dfrac{z-z(t_0)}{z'(t_0)}$

法平面方程 $x'(t_0)(x-x(t_0))+y'(t_0)(y-y(t_0))+z'(t_0)(z-z(t_0))=0$.

$s=\int_a^b|\vec{r}'(x)|\,dx=\int_a^b\sqrt{1+f'^2(x)}\,dx$

$s(t)=\int_a^t|\vec{r}'(u)|\,du$.　$\dfrac{ds}{dt}=|\vec{r}'(t)|=\sqrt{x'^2(t)+y'^2(t)+z'^2(t)}$

曲率 $k=\dfrac{\vec{r}'(t)\times\vec{r}''(t)}{|\vec{r}'(t)|^3}$.

10. 空间参数式曲面. $\begin{cases}x=x(u,v)\\ y=y(u,v)\\ z=z(u,v)\end{cases}$, $(u,v)\in D$.

$\dfrac{\partial\vec{r}}{\partial u}=\dfrac{\partial x}{\partial u}\vec{i}+\dfrac{\partial y}{\partial u}\vec{j}+\dfrac{\partial z}{\partial u}\vec{k}$

$\dfrac{\partial\vec{r}}{\partial v}=\dfrac{\partial x}{\partial v}\vec{i}+\dfrac{\partial y}{\partial v}\vec{j}+\dfrac{\partial z}{\partial v}\vec{k}$

$\Rightarrow d\vec{r}=dx(u,v)\vec{i}+dy(u,v)\vec{j}+dz(u,v)\vec{k}$.

$=\vec{r}_u'du+\vec{r}_v'dv$.

法向量 $\vec{n}(u,v)=\vec{r}_u'(u,v)\times\vec{r}_v'(u,v)$.

$=\dfrac{\partial(y,z)}{\partial(u,v)}\vec{i}+\dfrac{\partial(z,x)}{\partial(u,v)}\vec{j}+\dfrac{\partial(x,y)}{\partial(u,v)}\vec{k}$.

(曲线 $\begin{cases}x(t)=x(u(t),v(t))\\ y(t)=y(u(t),v(t))\\ z(t)=z(u(t),v(t))\end{cases}$ 切向量 $\vec{r}_u(u_0,v_0)u'(t_0)+\vec{r}_v(u_0,v_0)v'(t_0)$)

切平面 $\dfrac{\partial(y,z)}{\partial(u,v)}(X-x(u,v))+\dfrac{\partial(z,x)}{\partial(u,v)}(Y-y(u,v))$

$+\dfrac{\partial(x,y)}{\partial(u,v)}(Z-z(u,v))=0$.

11. 空间一般曲面. $F(x,y,z)\to F(x,y,z)=0$.

曲面上一点 $M(x_0,y_0,z_0)$ ($F(x_0,y_0,z_0)=0$).

曲面上的曲线 $\vec{r}(t)=x(t)\vec{i}+y(t)\vec{j}+z(t)\vec{k}$.

(满足 $F(x(t),y(t),z(t))=0$)

求导得 $\dfrac{\partial F(x,y,z)}{\partial x}x'(t)+\dfrac{\partial F(x,y,z)}{\partial y}y'(t)+\dfrac{\partial F}{\partial z}z'(t)=0$.

过该点 $\dfrac{\partial F(x_0,y_0,z_0)}{\partial x}x'(t_0)+\dfrac{\partial F}{\partial y}y'(t_0)+\dfrac{\partial F}{\partial z}z'(t_0)=0$.

法向量 $\vec{n}(x_0,y_0,z_0)=F_x'(x_0,y_0,z_0)\vec{i}+F_y'\vec{j}+F_z'\vec{k}$

切平面 $F_x'(x_0,y_0,z_0)(X-x_0)+F_y'(Y-y_0)+F_z'(Z-z_0)=0$.

12. 空间一般曲线

$\begin{cases}F(x,y,z)=0\\ G(x,y,z)=0\end{cases}$　曲面交线)

法向量 $\begin{cases}\vec{n}_1(x_0,y_0,z_0)=F_x'\vec{i}+F_y'\vec{j}+F_z'\vec{k}\\ \vec{n}_2(x_0,y_0,z_0)=G_x'\vec{i}+G_y'\vec{j}+G_z'\vec{k}\end{cases}$.

$\vec{n}_1\times\vec{n}_2=\dfrac{\partial(F,G)}{\partial(y,z)}\vec{i}+\dfrac{\partial(F,G)}{\partial(z,x)}\vec{j}+\dfrac{\partial(F,G)}{\partial(x,y)}\vec{k}$.