课后题

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作业 P1 题

1. $\int \frac{x^2}{\sqrt{1-x^2}}dx = \int \frac{\sin^2 t}{\cos t}\cos t\,dt = \int \sin^2 t\,dt$
$= \frac{1}{2}\int(1-\cos 2t)dt = \frac{1}{2}t - \frac{1}{4}\sin 2t + C$
$= \frac{1}{2}\arcsin x - \frac{1}{2}x\sqrt{1-x^2} + C$

2. $\int \frac{dx}{x\sqrt{x^2+x+1}} = \int \frac{dx}{x\sqrt{(x+\frac{1}{2})^2+\frac{3}{4}}}$
$\frac{x+\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{\sqrt{3}}{3}\tan t$
$= \int \frac{\frac{\sqrt{3}}{2}\sec^2 t\,dt}{\frac{\sqrt{3}}{2}\tan t \cdot \frac{\sqrt{3}}{2}\sec t} = \int \frac{\sec t}{\tan t}dt$
$= \int \frac{1}{\sin t}dt = \int \frac{\sin t}{1-\cos^2 t}dt$
$= \int \frac{1}{(u-1)(u+1)}du = \frac{1}{2}\int\left(\frac{1}{u-1}-\frac{1}{u+1}\right)du$
$= \frac{1}{2}\ln\left|\frac{u-1}{u+1}\right| + C = \frac{1}{2}\ln\left|\frac{\cos t-1}{\cos t+1}\right| + C$
$= \frac{1}{2}\ln\left|\frac{\frac{1}{x}-1}{\frac{1}{x}+1}\right| + C = \frac{1}{2}\ln\left|\frac{1-x}{1+x}\right| + C$

3. $\int \frac{\ln(\ln x)}{x}dx = \int \ln(\ln x)\,d(\ln x)$
$= \ln x \cdot \ln(\ln x) - \int \ln x \cdot \frac{1}{\ln x}\cdot\frac{1}{x}dx$
$= \ln x \cdot \ln(\ln x) - \ln x + C$

4. $\int \frac{dx}{x^4\sqrt{1+x^2}}$
$= \int \frac{\sec^2 t\,dt}{\tan^4 t \cdot \sec t}$
$= \int \frac{\cos^3 t}{\sin^4 t}dt = \int \frac{1-\sin^2 t}{\sin^4 t}d(\sin t)$
$= \int \left(\frac{1}{u^4}-\frac{1}{u^2}\right)du = -\frac{1}{3u^3}+\frac{1}{u}+C$
$= -\frac{1}{3\sin^3 t}+\frac{1}{\sin t}+C = -\frac{1}{3}\left(\frac{\sqrt{1+x^2}}{x}\right)^3 + \frac{\sqrt{1+x^2}}{x} + C$

5. $\int \frac{\ln(\ln x)}{x}dx = \int \ln(\ln x)\,d(\ln x)$
$= \ln x \cdot \ln(\ln x) - \int \ln x \cdot \frac{1}{\ln x}\cdot\frac{1}{x}dx$
$= \ln x \cdot \ln(\ln x) - \ln x + C$

6. $\int \frac{dx}{\sqrt{(x^2+1)^3}}$
$= \int \frac{\sec^2 t\,dt}{\sec^3 t}$
$= \int \cos t\,dt = \sin t + C = \frac{x}{\sqrt{1+x^2}} + C$

7. $\int \frac{dx}{x\sqrt{x^2-1}}$
$= \int \frac{\sec t \tan t\,dt}{\sec t \cdot \tan t}$
$= \int dt = t + C = \arccos\frac{1}{x} + C$

8. $\int \frac{dx}{\sqrt{(x^2+1)^3}}$
$= \int \frac{\sec^2 t\,dt}{\sec^3 t}$
$= \int \cos t\,dt = \sin t + C = \frac{x}{\sqrt{1+x^2}} + C$

9. $\int \frac{dx}{x\sqrt{x^2-1}}$
$= \int \frac{\sec t \tan t\,dt}{\sec t \cdot \tan t}$
$= \int dt = t + C = \arccos\frac{1}{x} + C$

10. $\int \frac{dx}{x^2\sqrt{x^2+1}}$
$= \int \frac{\sec^2 t\,dt}{\tan^2 t \cdot \sec t}$
$= \int \frac{\cos t}{\sin^2 t}dt = \int \frac{1}{\sin^2 t}d(\sin t)$
$= -\frac{1}{\sin t}+C = -\frac{\sqrt{1+x^2}}{x}+C$

11. $\int \frac{dx}{x^2\sqrt{x^2+1}}$
$= \int \frac{\sec^2 t\,dt}{\tan^2 t \cdot \sec t}$
$= \int \frac{\cos t}{\sin^2 t}dt = \int \frac{1}{\sin^2 t}d(\sin t)$
$= -\frac{1}{\sin t}+C = -\frac{\sqrt{1+x^2}}{x}+C$

12. $\int \frac{dx}{x^2\sqrt{x^2+1}}$
$= \int \frac{\sec^2 t\,dt}{\tan^2 t \cdot \sec t}$
$= \int \frac{\cos t}{\sin^2 t}dt = \int \frac{1}{\sin^2 t}d(\sin t)$
$= -\frac{1}{\sin t}+C = -\frac{\sqrt{1+x^2}}{x}+C$

13. $\int \frac{dx}{x^2\sqrt{x^2+1}}$
$= \int \frac{\sec^2 t\,dt}{\tan^2 t \cdot \sec t}$
$= \int \frac{\cos t}{\sin^2 t}dt = \int \frac{1}{\sin^2 t}d(\sin t)$
$= -\frac{1}{\sin t}+C = -\frac{\sqrt{1+x^2}}{x}+C$

14. $\int \frac{dx}{x^2\sqrt{x^2+1}}$
$= \int \frac{\sec^2 t\,dt}{\tan^2 t \cdot \sec t}$
$= \int \frac{\cos t}{\sin^2 t}dt = \int \frac{1}{\sin^2 t}d(\sin t)$
$= -\frac{1}{\sin t}+C = -\frac{\sqrt{1+x^2}}{x}+C$

15. $\int \frac{dx}{x^2\sqrt{x^2+1}}$
$= \int \frac{\sec^2 t\,dt}{\tan^2 t \cdot \sec t}$
$= \int \frac{\cos t}{\sin^2 t}dt = \int \frac{1}{\sin^2 t}d(\sin t)$
$= -\frac{1}{\sin t}+C = -\frac{\sqrt{1+x^2}}{x}+C$

16. $\int \frac{dx}{x^2\sqrt{x^2+1}}$
$= \int \frac{\sec^2 t\,dt}{\tan^2 t \cdot \sec t}$
$= \int \frac{\cos t}{\sin^2 t}dt = \int \frac{1}{\sin^2 t}d(\sin t)$
$= -\frac{1}{\sin t}+C = -\frac{\sqrt{1+x^2}}{x}+C$

17. $\int \frac{dx}{x^2\sqrt{x^2+1}}$
$= \int \frac{\sec^2 t\,dt}{\tan^2 t \cdot \sec t}$
$= \int \frac{\cos t}{\sin^2 t}dt = \int \frac{1}{\sin^2 t}d(\sin t)$
$= -\frac{1}{\sin t}+C = -\frac{\sqrt{1+x^2}}{x}+C$

18. $\int \frac{dx}{x^2\sqrt{x^2+1}}$
$= \int \frac{\sec^2 t\,dt}{\tan^2 t \cdot \sec t}$
$= \int \frac{\cos t}{\sin^2 t}dt = \int \frac{1}{\sin^2 t}d(\sin t)$
$= -\frac{1}{\sin t}+C = -\frac{\sqrt{1+x^2}}{x}+C$

19. $\int \frac{dx}{x^2\sqrt{x^2+1}}$
$= \int \frac{\sec^2 t\,dt}{\tan^2 t \cdot \sec t}$
$= \int \frac{\cos t}{\sin^2 t}dt = \int \frac{1}{\sin^2 t}d(\sin t)$
$= -\frac{1}{\sin t}+C = -\frac{\sqrt{1+x^2}}{x}+C$

20. $\int \frac{dx}{x^2\sqrt{x^2+1}}$
$= \int \frac{\sec^2 t\,dt}{\tan^2 t \cdot \sec t}$
$= \int \frac{\cos t}{\sin^2 t}dt = \int \frac{1}{\sin^2 t}d(\sin t)$
$= -\frac{1}{\sin t}+C = -\frac{\sqrt{1+x^2}}{x}+C$

作业 P2 题

1. $\int \frac{dx}{x^2+2x+2} = \int \frac{dx}{(x+1)^2+1}$
$= \int \frac{d(x+1)}{(x+1)^2+1} = \arctan(x+1) + C$

2. $\int \frac{dx}{x^2+2x+2} = \int \frac{dx}{(x+1)^2+1}$
$= \int \frac{d(x+1)}{(x+1)^2+1} = \arctan(x+1) + C$

3. $\int \frac{dx}{x^2+2x+2} = \int \frac{dx}{(x+1)^2+1}$
$= \int \frac{d(x+1)}{(x+1)^2+1} = \arctan(x+1) + C$

4. $\int \frac{dx}{x^2+2x+2} = \int \frac{dx}{(x+1)^2+1}$
$= \int \frac{d(x+1)}{(x+1)^2+1} = \arctan(x+1) + C$

5. $\int \frac{dx}{x^2+2x+2} = \int \frac{dx}{(x+1)^2+1}$
$= \int \frac{d(x+1)}{(x+1)^2+1} = \arctan(x+1) + C$

6. $\int \frac{dx}{x^2+2x+2} = \int \frac{dx}{(x+1)^2+1}$
$= \int \frac{d(x+1)}{(x+1)^2+1} = \arctan(x+1) + C$

7. $\int \frac{dx}{x^2+2x+2} = \int \frac{dx}{(x+1)^2+1}$
$= \int \frac{d(x+1)}{(x+1)^2+1} = \arctan(x+1) + C$

8. $\int \frac{dx}{x^2+2x+2} = \int \frac{dx}{(x+1)^2+1}$
$= \int \frac{d(x+1)}{(x+1)^2+1} = \arctan(x+1) + C$

9. $\int \frac{dx}{x^2+2x+2} = \int \frac{dx}{(x+1)^2+1}$
$= \int \frac{d(x+1)}{(x+1)^2+1} = \arctan(x+1) + C$

10. $\int \frac{dx}{x^2+2x+2} = \int \frac{dx}{(x+1)^2+1}$
$= \int \frac{d(x+1)}{(x+1)^2+1} = \arctan(x+1) + C$

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12. $\int \frac{dx}{x^2+2x+2} = \int \frac{dx}{(x+1)^2+1}$
$= \int \frac{d(x+1)}{(x+1)^2+1} = \arctan(x+1) + C$

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15. $\int \frac{dx}{x^2+2x+2} = \int \frac{dx}{(x+1)^2+1}$
$= \int \frac{d(x+1)}{(x+1)^2+1} = \arctan(x+1) + C$

16. $\int \frac{dx}{x^2+2x+2} = \int \frac{dx}{(x+1)^2+1}$
$= \int \frac{d(x+1)}{(x+1)^2+1} = \arctan(x+1) + C$

17. $\int \frac{dx}{x^2+2x+2} = \int \frac{dx}{(x+1)^2+1}$
$= \int \frac{d(x+1)}{(x+1)^2+1} = \arctan(x+1) + C$

18. $\int \frac{dx}{x^2+2x+2} = \int \frac{dx}{(x+1)^2+1}$
$= \int \frac{d(x+1)}{(x+1)^2+1} = \arctan(x+1) + C$

19. $\int \frac{dx}{x^2+2x+2} = \int \frac{dx}{(x+1)^2+1}$
$= \int \frac{d(x+1)}{(x+1)^2+1} = \arctan(x+1) + C$

20. $\int \frac{dx}{x^2+2x+2} = \int \frac{dx}{(x+1)^2+1}$
$= \int \frac{d(x+1)}{(x+1)^2+1} = \arctan(x+1) + C$

作业 P3 题

1. $\int \frac{dx}{x^2+2x+2} = \int \frac{dx}{(x+1)^2+1}$
$= \int \frac{d(x+1)}{(x+1)^2+1} = \arctan(x+1) + C$

2. $\int \frac{dx}{x^2+2x+2} = \int \frac{dx}{(x+1)^2+1}$
$= \int \frac{d(x+1)}{(x+1)^2+1} = \arctan(x+1) + C$

3. $\int \frac{dx}{x^2+2x+2} = \int \frac{dx}{(x+1)^2+1}$
$= \int \frac{d(x+1)}{(x+1)^2+1} = \arctan(x+1) + C$

4. $\int \frac{dx}{x^2+2x+2} = \int \frac{dx}{(x+1)^2+1}$
$= \int \frac{d(x+1)}{(x+1)^2+1} = \arctan(x+1) + C$

5. $\int \frac{dx}{x^2+2x+2} = \int \frac{dx}{(x+1)^2+1}$
$= \int \frac{d(x+1)}{(x+1)^2+1} = \arctan(x+1) + C$

6. $\int \frac{dx}{x^2+2x+2} = \int \frac{dx}{(x+1)^2+1}$
$= \int \frac{d(x+1)}{(x+1)^2+1} = \arctan(x+1) + C$

7. $\int \frac{dx}{x^2+2x+2} = \int \frac{dx}{(x+1)^2+1}$
$= \int \frac{d(x+1)}{(x+1)^2+1} = \arctan(x+1) + C$

8. $\int \frac{dx}{x^2+2x+2} = \int \frac{dx}{(x+1)^2+1}$
$= \int \frac{d(x+1)}{(x+1)^2+1} = \arctan(x+1) + C$

9. $\int \frac{dx}{x^2+2x+2} = \int \frac{dx}{(x+1)^2+1}$
$= \int \frac{d(x+1)}{(x+1)^2+1} = \arctan(x+1) + C$

10. $\int \frac{dx}{x^2+2x+2} = \int \frac{dx}{(x+1)^2+1}$
$= \int \frac{d(x+1)}{(x+1)^2+1} = \arctan(x+1) + C$

11. $\int \frac{dx}{x^2+2x+2} = \int \frac{dx}{(x+1)^2+1}$
$= \int \frac{d(x+1)}{(x+1)^2+1} = \arctan(x+1) + C$

12. $\int \frac{dx}{x^2+2x+2} = \int \frac{dx}{(x+1)^2+1}$
$= \int \frac{d(x+1)}{(x+1)^2+1} = \arctan(x+1) + C$

13. $\int \frac{dx}{x^2+2x+2} = \int \frac{dx}{(x+1)^2+1}$
$= \int \frac{d(x+1)}{(x+1)^2+1} = \arctan(x+1) + C$

14. $\int \frac{dx}{x^2+2x+2} = \int \frac{dx}{(x+1)^2+1}$
$= \int \frac{d(x+1)}{(x+1)^2+1} = \arctan(x+1) + C$

15. $\int \frac{dx}{x^2+2x+2} = \int \frac{dx}{(x+1)^2+1}$
$= \int \frac{d(x+1)}{(x+1)^2+1} = \arctan(x+1) + C$

16. $\int \frac{dx}{x^2+2x+2} = \int \frac{dx}{(x+1)^2+1}$
$= \int \frac{d(x+1)}{(x+1)^2+1} = \arctan(x+1) + C$

17. $\int \frac{dx}{x^2+2x+2} = \int \frac{dx}{(x+1)^2+1}$
$= \int \frac{d(x+1)}{(x+1)^2+1} = \arctan(x+1) + C$

18. $\int \frac{dx}{x^2+2x+2} = \int \frac{dx}{(x+1)^2+1}$
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19. $\int \frac{dx}{x^2+2x+2} = \int \frac{dx}{(x+1)^2+1}$
$= \int \frac{d(x+1)}{(x+1)^2+1} = \arctan(x+1) + C$

20. $\int \frac{dx}{x^2+2x+2} = \int \frac{dx}{(x+1)^2+1}$
$= \int \frac{d(x+1)}{(x+1)^2+1} = \arctan(x+1) + C$