知识点

2022年5月9日 星期一　上午7:21

1. 求Fourier级数.

周期为2π的某函数$f(x)$可以展成Fourier级数.

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}(a_n\cos nx+b_n\sin nx).$$

若$f(x)$分段可微，则该级数收敛于$f(x)$

若$f(x)$处处连续，则该级数一致收敛于$f(x)$.

因此计算系数：$a_m=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\cos mx\,dx$.

$b_m=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\sin mx\,dx$.

若周期不是2π而为$2l$，进行$t=\frac{\pi}{l}x$的替换，

$$a_n=\frac{1}{\pi}\int_{-\pi}^{\pi}g(t)\cos nt\,dt=\frac{1}{l}\int_{-l}^{l}f(x)\cos\frac{n\pi x}{l}\,dx$$

$$b_n=\frac{1}{\pi}\int_{-\pi}^{\pi}g(t)\sin nt\,dt=\frac{1}{l}\int_{-l}^{l}f(x)\sin\frac{n\pi x}{l}\,dx$$

2. 平方平均收敛.

Parseval等式：$\frac{1}{\pi}\int_{-\pi}^{\pi}f^2(t)\,dt=\frac{a_0^2}{2}+\sum_{n=1}^{\infty}(a_n^2+b_n^2)$

正交. 线性性. 部分和.

3.

$$\frac{1}{2\pi}\int_{-\infty}^{+\infty}d\lambda\, e^{i\lambda x}\int_{-\infty}^{+\infty}d\xi\, f(\xi)e^{-i\lambda\xi}=\frac{f(x+0)+f(x-0)}{2}.$$

4. Fourier变换.

$$F(\lambda)=\int_{-\infty}^{+\infty}f(\xi)e^{-i\lambda\xi}\,d\xi,$$

$$f(x)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}F(\lambda)e^{i\lambda x}\,d\lambda.$$

① 如果$f(x)$为偶，则$f(x)\sin\lambda x$为奇，

$$f(x)\to F(\lambda)=\int_{-\infty}^{+\infty}f(\xi)e^{-i\lambda\xi}\,d\xi$$

$$=\int_{-\infty}^{+\infty}f(t)(\cos\lambda t-i\sin\lambda t)\,dt$$

$$=2\int_{0}^{+\infty}f(t)\cos\lambda t\,dt$$ 余弦变换

$$F(\lambda)\to f(x)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}F(\lambda)(\cos\lambda x+i\sin\lambda x)\,d\lambda$$

$$=\frac{1}{\pi}\int_{0}^{+\infty}F(\lambda)\cos\lambda x\,d\lambda.$$

② 如果$f(x)$为奇函数，则

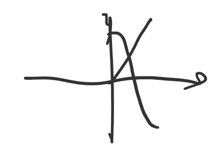
$$f(x)\to F(\lambda)=-2i\int_{0}^{+\infty}f(t)\sin\lambda t\,dt$$

令 $G(\lambda)=iF(\lambda)=2\int_{0}^{\infty}f(t)\sin\lambda t\,dt$ 正弦变换.

$$G(\lambda)\to f(x)=\frac{1}{\pi}\int_{0}^{+\infty}G(\lambda)\sin\lambda x\,d\lambda.$$

$$\int_0^\pi\cos nx\,dx=\int_0^\pi\frac{1}{n}\cos nx\,dnx=\frac{1}{n}\sin nx\Big|_0^\pi=\frac{1}{n}\sin n\pi=0$$

$$\int_0^\pi\sin nx\,dx=\int_0^\pi\frac{1}{n}\sin nx\,dnx=-\frac{1}{n}\cos nx\Big|_0^\pi=\frac{1-(-1)^n}{n}$$



$$\int_0^\pi x\cos nx\,dx=\int_0^\pi\frac{1}{n^2}nx\cos nx\,dnx=\frac{1}{n^2}\int_0^\pi nx\,d\sin nx=\frac{1}{n^2}\left(nx\sin nx\Big|_0^\pi-\int_0^\pi\sin nx\,dnx\right)=\frac{1}{n^2}(-1+(-1)^n)=\frac{(-1)^n-1}{n^2}$$

$$\int_0^\pi x\sin nx\,dx=\int_0^\pi\frac{1}{n^2}nx\sin nx\,dnx=\frac{1}{n^2}\int_0^\pi -nx\,d\cos nx=-\frac{1}{n^2}\left(nx\cos nx\Big|_0^\pi-\int_0^\pi\cos nx\,dnx\right)=-\frac{1}{n^2}((-1)^n n\pi-0)=\frac{(-1)^{n+1}\pi}{n}.$$

$$\int_0^\pi x^2\cos nx\,dx=\int_0^\pi\frac{1}{n^3}(nx)^2\cos nx\,dnx=\frac{1}{n^3}\left((nx)^2\sin nx\Big|_0^\pi-\int_0^\pi\sin nx\,d(nx)^2\right)=\frac{1}{n^3}\left(-2\int_0^\pi(nx)\sin nx\,dnx\right)=\frac{-2}{n^3}\left(-\int_0^\pi nx\,d\cos nx\right)=\frac{2}{n^3}\left(nx\cos nx\Big|_0^\pi-\int_0^\pi\cos nx\,dnx\right)=\frac{2}{n^3}(\pi(-1)^n-0)=\frac{(-1)^n2\pi}{n^2}.$$

$$\int_0^\pi x^2\sin nx\,dx=\int_0^\pi\frac{1}{n^3}(nx)^2\sin nx\,dnx=-\frac{1}{n^3}\left[(nx)^2\cos nx\Big|_0^\pi-\int_0^\pi\cos nx\,d(nx)^2\right]=-\frac{1}{n^3}\left(n^2\pi^2(-1)^n-2\int_0^\pi(nx)\cos nx\,dnx\right)=-\frac{1}{n}\left(\pi^2(-1)^n-2\int_0^\pi x\cos nx\,dx\right)=-\frac{1}{n}\left(\pi^2(-1)^n-\frac{2((-1)^n-1)}{n^2}\right)=\frac{2[(-1)^n-1]-n^2\pi^2(-1)^n}{n^3}.$$

$$\int t\cos t\,dt=\int t\,d\sin t=t\sin t-\int\sin t\,dt=t\sin t+\cos t$$

$$\int t\sin t\,dt=-\int t\,d\cos t=-t\cos t+\int\cos t\,dt=-t\cos t+\sin t$$

$$\int t^2\cos t\,dt=\left(t^2\sin t-2\int t\sin t\,dt\right)=t^2\sin t+2\int t\,d\cos t=t^2\sin t+2t\cos t-2\int\cos t\,dt=t^2\sin t+2t\cos t-2\sin t$$

$$\int t^2\sin t\,dt=t^2\cos t-2\int t\cos t\,dt=t^2\cos t-2\int t\,d\sin t=t^2\cos t-2t\sin t+2\int\sin t\,dt=t^2\cos t-2t\sin t-2\cos t\Big|_0^{n\pi}$$

$$\frac{(-1)^n n^2\pi^2-2(-1)^n}{n^3}$$