课后题

2022年5月8日 星期日　　下午10:35

习题 12.1

(1.1)

$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$

$= \frac{1}{\pi}\left(\int_{-\pi}^{0} -\pi\,dx + \int_0^{\pi} x\,dx\right)$

$= \frac{1}{\pi}\left(-\pi^2 + \frac{1}{2}\pi^2\right) = -\frac{\pi}{2}$

$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx$

$= \frac{1}{\pi}\left(\int_{-\pi}^{0} -\pi\cos nx\,dx + \int_0^{\pi} x\cos nx\,dx\right)$

$= \frac{1}{\pi}\left(0 + \frac{(-1)^n-1}{n^2}\right) = \frac{(-1)^n-1}{n^2\pi}$

$b_n = \frac{1}{\pi}\left(\int_{-\pi}^{0} -\pi\sin nx\,dx + \int_0^{\pi} x\sin nx\,dx\right)$

$= \frac{1}{\pi}\left(2\pi + \frac{(-1)^{n+1}\pi}{n}\right) = 2 + \frac{(-1)^{n+1}}{n}$

(2)

$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} \cos\frac{x}{2}\,dx$

$= 2\cdot\frac{1}{\pi}\int \cos\frac{x}{2}\,d\frac{x}{2}$

$= \frac{2}{\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos t\,dt = \frac{4}{\pi}$

$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} \cos\frac{x}{2}\cos nx\,dx = 0$

习题 12.2

1. $a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx = \frac{2}{\pi}\int_0^a 1\,dx = \frac{2a}{\pi}$.

$a_n = \frac{2}{\pi}\left(\int_0^a \cos nx\,dx\right) = \frac{2}{\pi n}\sin t\Big|_0^{na} = \frac{2\sin na}{\pi n}$.

$f(x) = \frac{a}{\pi} + \sum_{n=1}^{\infty}\frac{2\sin na}{\pi n}\cos nx$.

(1) $\frac{1}{\pi}\int_{-\pi}^{\pi} f^2(x)\,dx = \frac{a_0^2}{2} + \sum_{n=1}^{\infty}(a_n^2+b_n^2)$

(2) $f(x) = \frac{a}{\pi} + \sum_{n=1}^{\infty}\frac{2\cos nx}{\pi n}\sin na$. 与(1)同.

3. $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n\cos nx + b_n\sin nx$

$\frac{1}{\pi}\int_{-\pi}^{\pi} f^2(x)\,dx = \frac{a_0^2}{2} + \sum_{n=1}^{\infty}(a_n^2+b_n^2)$ 有限. $\sum a_n^2$ 收敛. 且 $\sum\frac{1}{n^2}$ 收敛.

习题 12.3

8. 奇延拓后

[illegible]

习题 12.4

[illegible]