补充题

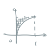
2022年4月3日 星期日　　下午4:30

学习指导.

二重.

59.(1) $\int_0^1dx\int_{\sqrt[3]{x}}^1e^{y^2}dy$

e^{y^2}不好算，要计算需先换积分次序.



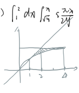
$\int_0^1e^{y^2}dy\int_0^{y^3}dx$

$=\int_0^1y^3e^{y^2}dy=\frac12\int_0^1y^2e^{y^2}dy^2$

$=\frac12\int_0^1te^tdt=\frac12\int_0^1t\,de^t$

$=\frac12(te^t|_0^1-\int_0^1e^tdt)=\frac12(e-e+1)=\frac12.$

(2) $\int_1^2dx\int_{\sqrt x}^x\sin\frac{\pi x}{2y}dy+\int_2^4dx\int_{\sqrt x}^2\sin\frac{\pi x}{2y}dy$



$\int_1^2dy\int_y^{y^2}\sin\frac{\pi x}{2y}dx$

$=\int_1^2dy\,\frac{2y}{\pi}(-\cos\frac{\pi x}{2y})|_y^{y^2}$

$=\int_1^2\frac{2y}{\pi}(\cos\frac{\pi}{2y}y-\cos\frac{\pi}{2y}y^2)dy$

$=\int_1^2\frac{2y}{\pi}(0-\cos\frac{\pi}{2}y)dy$

$=\frac{4}{\pi}\int_1^2\frac{\pi}{2}y\cos\frac{\pi}{2}y\,dy$

$=\frac{8}{\pi^3}\int_{\frac{\pi}{2}}^{\pi}\frac{\pi}{2}y\cos\frac{\pi}{2}y\,d\frac{\pi}{2}y$

$=\frac{8}{\pi^3}\int_{\frac{\pi}{2}}^{\pi}t\cos t\,dt$

$=-\frac{8}{\pi^3}(1+\frac{\pi}{2})$

$\int_{\frac{\pi}{2}}^{\pi}t\,d\sin t$

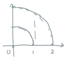
$=-\int_{\frac{\pi}{2}}^{\pi}\sin t\,dt+t\sin t$

$=(\cos t+t\sin t)|_{\frac{\pi}{2}}^{\pi}$

$=-1+\cancel{\pi\cdot0}-\frac{\pi}{2}\times1$

$=-1-\frac{\pi}{2}$

(3) $\int_0^1dx\int_{\sqrt{1-x^2}}^{\sqrt{4-x^2}}(5-x^2-y^2)^\alpha dy+\int_1^2dx\int_0^{\sqrt{4-x^2}}(5-x^2-y^2)^\alpha dy$



$x=r\cos\theta$

$y=r\sin\theta$.

$\int_1^2\int_0^{\frac{\pi}{2}}(5-r^2)^\alpha r\,d\theta\,dr$

$=\frac{\pi}{2}\int_1^2(5-r^2)^\alpha\cdot\frac12dr^2.$

62. $\lim_{\rho\to0}\frac{1}{\pi\rho^2}\iint_{x^2+y^2\le\rho^2}f(x,y)dxdy$.

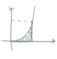
中值定理. $\exists(\xi,\eta)\in\{(x,y)|x^2+y^2\le\rho^2\}$

$\lim_{\rho\to0}\frac{1}{\pi\rho^2}\iint f(x,y)dxdy=\lim_{\rho\to0}\frac{1}{\pi\rho^2}f(\xi,\eta)\iint_{x^2+y^2\le\rho^2}dxdy.$

$=\lim_{(\xi,\eta)\to(0,0)}f(\xi,\eta)=f(0,0).$

63. $y=x^3$. $y=0$. $x=1$. 围成，求$f(x,y)$值

$f(x,y)=xy+\underbrace{\iint f(x,y)dxdy}_{g(x,y)}.$



$\frac{\partial^2g(x,y)}{\partial x\partial y}=xy+g(x,y).$

$\frac{\partial g(x,y)}{\partial x}=\frac12xy^2+\int g(x,y)dy+\varphi(y)$

$g(x,y)=\frac14x^2y^2+\iint_D g(x,y)dydx+\varphi(y)x+\psi(x)$

设 $\iint f(x,y)dxdy=A$.

$\iint f(x,y)dxdy=\iint xy\,dxdy+\iint_D(\iint_D f(x,y)dxdy)dxdy.$

$A=\int_0^1\int_0^{x^3}xy\,dydx+\int_0^1\int_0^{x^3}A\,dxdy.$

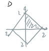
$=\int_0^1x\cdot\frac12y^2|_0^{x^3}dx+\cdots=\int_0^1\frac12x^7dx+\cdots$

$=\frac1{16}x^8|_0^1dx+\int_0^1\int_0^{x^3}A\,dxdy$　A常数.

$=\frac1{16}+\frac14A.$

$\therefore A=\frac1{12}=\iint_Df(x,y)dxdy$. $\therefore f(x,y)=xy+\frac1{12}.$

64.(1) $\iint_D(|x|+|y|)dxdy$　$\{(x,y)||x|+|y|\le2\}$.



$4\int_0^2dx\int_0^{2-x}x+y\,dy.$

(2) $\iint_D\frac{af(x)+bf(y)}{f(x)+f(y)}dxdy$　$\{(x,y)||x|+|y|\le1\}$.

$\frac{af(x)+af(y)+(b-a)f(y)}{f(x)+f(y)}=a+\frac{(b-a)f(y)}{f(x)+f(y)}$

$=b+\frac{(a-b)f(x)}{f(x)+f(y)}$

关于$y=x$对称.

$\iint_D\frac{af(x)+bf(y)}{f(x)+f(y)}dxdy=\iint_D\frac{af(y)+bf(x)}{f(y)+f(x)}dxdy.$

$=\frac12\iint_D\frac{(a+b)f(x)+(a+b)f(y)}{f(x)+f(y)}dxdy.$

$=\frac12(a+b)\iint_Ddxdy=a+b.$

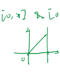
(3) $\iint_D\frac{dxdy}{(1+x^2+y^2)^{\frac32}}$. $\{(x,y)|0\le x\le1,0\le y\le1\}$.

$x=r\cos\theta$　$y=r\sin\theta$　$\int_0^1\int_0^{\frac{\pi}{2}}(1+r^2)^{-\frac32}r\,d\theta\,dr=\frac{\pi}{2}\times\frac12\int_0^1(r^2+1)^{\frac32}d(r^2+1)$

$=\frac{\pi}{2}\times\frac25(r^2+1)^{\frac52}|_0^1=\frac{\pi}{10}(2^{\frac52}-1)=\frac{\pi}{10}(4\sqrt2-1)$

D关于$y=x$对称. $[0,1]$. $[0,x]$ & $[0,1]$.$[x,1]$.

$\iint_D\frac{dxdy}{(1+x^2+y^2)^{\frac32}}=2\int_0^1\int_0^x\frac{dxdy}{(1+x^2+y^2)^{\frac32}}$



$x=r\cos\theta$　$x=1$, $r=\frac{1}{\cos\theta}$

$y=r\sin\theta$

$\int_0^{\frac{\pi}{4}}\int_0^{\frac{1}{\cos\theta}}\frac{\rho\,dr\,d\theta}{(1+r^2)^{\frac32}}=\int_0^{\frac{\pi}{4}}\int_0^{\frac{1}{\cos\theta}}\frac{\frac12dr^2d\theta}{(r^2+1)^{\frac32}}$

$=\frac12\int_0^{\frac{\pi}{4}}-\frac{2}{(r^2+1)^{\frac12}}|_0^{\frac1{\cos\theta}}d\theta=-\frac14\int_0^{\frac{\pi}{4}}\frac{\cos\theta}{\sqrt{\cos^2\theta+1}}-1\,d\theta$

$=\frac14\int_0^{\frac{\pi}{4}}\frac{1}{\cos^2\theta+1}d\theta=\frac14\int_0^{\frac{\pi}{4}}\frac{\cos^2\theta+\sin^2\theta}{2\cos^2\theta+\sin^2\theta}=\frac{1+\tan^2\theta}{2+\tan^2\theta}$

$=\frac14\int_0^{\frac{\pi}{4}}\frac12+\frac{\frac12\tan^2\theta}{2+\tan^2\theta}d\theta.$

$=-2\int_0^{\frac{\pi}{4}}(1-\frac{\cos\theta}{\sqrt{1+\cos^2\theta}})d\theta=\frac{\pi}{2}-2\arcsin\frac{\cos\theta}{\sqrt2}|_0^{\frac{\pi}{4}}=\frac{\pi}{6}.$

(4) $\iint_D|3x+4y|dxdy$. $D=\{(x,y)|x^2+y^2\le1\}$.

$x\ge0$, $y\ge0$.　$\int_0^1\int_0^{\sqrt{1-x^2}}3x+4y\,dy\,dx$　$\Rightarrow\frac43$

$=\int_0^13xy|_0^{\sqrt{1-x^2}}+2y^2|_0^{\sqrt{1-x^2}}dx$　坐标变换. $x=r\cos\theta$ $y=r\sin\theta$.

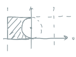
$=\int_0^13x\sqrt{1-x^2}+2(1-x^2)dx$

$=\int_0^1\frac32\sqrt{1-x^2}dx^2+2\int_0^1(1-x^2)dx$

$=-\frac32\times\frac23(1-x^2)^{\frac32}|_0^1+2(x-\frac13x^3)|_0^1$

$=(x^2-1)^{\frac32}|_0^1+2(1-\frac13)=-1^{\frac32}+\frac43=\frac13.$

(5) $\iint_Dy\,dxdy$. $x=-2$, $y=0$, $y=2$. $x=-\sqrt{2y-y^2}$.



$x^2=2y-y^2$　$x^2+y^2-2y=0$

$x^2+(y-1)^2=1$.

$\iint y\,dxdy=\int_0^2\int_{-2}^0y\,dxdy-\int_0^2\int_0^{\sqrt{2y-y^2}}y\,dxdy$

$=\int_0^2y\times2dy-\int_0^2y\sqrt{2y-y^2}dy.$

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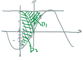
$\int_0^2y\sqrt{1-(y-1)^2}dy$　$(y=1-\cos t)$

$=\int_0^{\pi}(1-\cos t)\sin^2t\,dt$

$=4-\frac{\pi}{2}$

(6) $\iint_Dx(1+ye^{x^2y^6})dxdy$. $y=\sin x$. $x=-\frac{\pi}{2}$. $y=1$.



$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_{\sin x}^1x(1+ye^{x^2y^6})dxdy$

$=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_{\sin x}^1x+xe^{x^2}ye^{y^6}dydx$

两块对称区域. $xye^{x^2y^2}$奇函数.

$=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}xy|_{\sin x}^1+\underline{\int xe^{x^2}\frac12e^{y^6}dy^2}dx$

$\iint_{D_1}xye^{x^2y^6}dxdy+\iint_{D_2}xye^{x^2}e^{y^6}dxdy=0.$

$\frac12xe^{x^2}\int_{\sin x}^1e^{y^6}dy^2$

$=\int_{\sin x}^1e^{t^3}dt.$

$\iint_Dx(1+ye^{x^2y^2})dxdy$

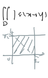
$=\iint_Dx\,dxdy=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}x\underline{\int_{\sin x}^1dy}dx$　$=\iint_{D_1}x\,dxdy$

D_1关于x对称

$=(1-\sin x)x|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}=\cancel{x}(1-1)\frac{\pi}{2}-(1+1)\frac{\pi}{2}$.

$=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}x(1-\sin x)dx=-2$

(7) $\iint_D|\sin(x+y)|dxdy$　$\{(x,y)|0\le x\le\pi,0\le y\le\pi\}$.



取$x+y=\pi$. $\sin(x+y)\ge0$　面$=2\pi$.

$\int_0^{\pi}\int_0^{\pi-x}\sin(x+y)dy\,dx$

$=\int_0^{\pi}-\cos(x+\pi-x)+\cos x\,dx$

$=\int_0^{\pi}-\cos\pi+\cos x\,dx$

$=(x+\sin x)|_0^{\pi}=\pi$

(8) $\iint_Dxy[1+x^2+y^2]dxdy$. $D=\{(x,y)|x^2+y^2\le\sqrt2, x\ge0, y\ge0\}$.

不超过的最大整数.

$\begin{cases}x=r\cos\theta\\y=r\sin\theta\end{cases}$　$r\in[0,\sqrt2]$. $\theta\in[0,\frac{\pi}{2}]$.

$\int_0^{\sqrt2}\int_0^{\frac{\pi}{2}}r^2\cos\theta\sin\theta[1+r^2]r\,d\theta\,dr.$

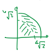
$[1+x^2+y^2]=\begin{cases}1+[x^2+y^2]=1\\2\end{cases}$

$\int_0^{\frac{\pi}{2}}\frac12\sin2\theta\int_0^{\sqrt2}r^3[1+r^2]dr\,d\theta.$

$0\le x^2+y^2<1$

$1\le x^2+y^2\le\sqrt2$.

$=-\frac14\cos2\theta|_0^{\frac{\pi}{2}}[\frac15r^5+\frac16r^6]|_0^{\sqrt2}.$



分块. $\iint_{D_1}xy\,dxdy+2\iint_{D_2}xy\,dxdy.$

65. $D=\{(x,y)|x^2+y^2\le t^2,t>0\}$.

(1) $f(t^2)=\iint_D(y^2+f(x^2+y^2))dxdy$, $f(1)=\pi$.

$\int_0^1f(x)dx$. $f(0)$.

$t=1$, $f(1)=\iint_{x^2+y^2\le1}y^2+f(x^2+y^2)dxdy=\pi$

区域关于$y=x$对称.

$\iint_Dy^2dxdy=\frac12\iint_D(x^2+y^2)dxdy=\frac12\int_0^{2\pi}d\theta\int_0^tr^3dr=\frac{\pi}{4}t^4.$

$\iint_Df(x^2+y^2)dxdy=\int_0^{2\pi}d\theta\int_0^trf(r^2)dr\ 2\pi=\pi f(u)du.$

$f(t^2)=\frac{\pi}{4}t^4+\pi\int_0^{t^2}f(u)du.$

$t=1$, $f(1)=\frac{\pi}{4}+\pi\int_0^1f(x)dx=\pi$.

$t=0$. $f(0)=0$

(2) $f'(0)=1$, $\lim_{t\to0^+}\frac{1}{\pi t^4}\iint_D(y^2+f(x^2+y^2))dxdy.$

$\lim_{t\to0^+}\frac{1}{\pi t^4}f(t^2)=\lim_{t\to0^+}\frac{\frac{\pi}{4}t^4+\pi\int_0^{t^2}f(u)du}{\pi t^4}$

$=\lim_{t\to0}\frac{\pi t^3+f(t^2)2t}{4\pi t^3}=\lim_{t\to0}\frac{t^2+2f(t^2)}{4t^2}=\cancel{\frac{1}{4}}$

$\frac34$　$\frac{f(t^2)}{2t^2}=\frac{f(t^2)-f(0)}{2(t^2-0)}=1.$

66. $\int_\alpha^\beta f(\cos\theta,\sin\theta)d\theta=A$, $f(0,0)=1$.

$D=\{(r,\theta)|0\le r\le1,\alpha\le\theta\le\beta,\beta-\alpha\le2\pi\}$.

$\iint(\frac{x}{x^2+y^2}\frac{\partial f}{\partial x}+\frac{y}{x^2+y^2}\frac{\partial f}{\partial y})dxdy$

$x=r\cos\theta$, $y=r\sin\theta$.

$\int_0^1\int_\alpha^\beta(\frac{r\cos\theta}{r^2}\frac{\partial f}{\partial x}+\frac{r\sin\theta}{r^2}\frac{\partial f}{\partial y})r\,dr\,d\theta$

$=\int_0^1\int_\alpha^\beta(\cos\theta\frac{\partial f}{\partial x}+\sin\theta\frac{\partial f}{\partial y})dr\,d\theta.$

$\frac{\partial}{\partial r}f(r\cos\theta,r\sin\theta)dr=\cos\theta\frac{\partial f}{\partial x}+\sin\theta\frac{\partial f}{\partial y}.$

$=\int_0^1\frac{\partial f}{\partial r}(r\cos\theta,r\sin\theta)dr\,d\theta$

$=\int_\alpha^\beta f(\cos\theta,\sin\theta)-f(0,0)d\theta=A-(\beta-\alpha).$

67.(1) $\iint_D\sqrt{x^2+y^2}dxdy$, $D=(x^2+y^2)^{\frac32}=y^2$.

$x=r\cos\theta$　$(r^2)^{\frac32}=y^2$. $r^3=r^2\sin^2\theta$, $r=\sin^2\theta$

$y=r\sin\theta$.　$r>0$. $0\le r\le\sin^2\theta$.

$2\int_0^{\pi}\int_0^{\sin\theta}r^2dr\,d\theta=2\int_0^{\pi}\frac13r^3|_0^{\sin^2\theta}d\theta=4\int_0^{\frac{\pi}{2}}\frac13\sin^6\theta\,d\theta=\frac{5}{24}\pi$

(2) $D:r=a(1+\cos\theta)$

$x=r\cos\theta$　$\int_0^{\pi}\int_0^{a(1+\cos\theta)}r^2dr\,d\theta$

$y=r\sin\theta$　$=\int_0^{\pi}\frac13a^3(1+\cos\theta)^3d\theta$

$=\frac{a^3}{3}(1+\cos^3\theta+3\cos^2\theta+3\cos\theta)d\theta=\cdots$

68.(1) $\iint_D\sqrt{4-x^2-4y^2}dxdy$. $x^2+4y^2\le2x$.

$x=r\cos\theta$, $y=\frac12r\sin\theta$.　$(x-1)^2+(2y)^2\le1$.

$r^2\cos^2\theta+4\times\frac14r^2\sin^2\theta=r^2\le2r\cos\theta$. $r\le2\cos\theta$.

$\int_0^{2\pi}\int_0^{2\cos\theta}\sqrt{4-r^2}\frac12r\,dr\,d\theta=\int_0^{\pi}\int_0^{2\cos\theta}\sqrt{4-r^2}\frac14dr^2d\theta$

$=\int_0^{2\pi}-\frac23(4-r^2)^{\frac32}\times\frac14|_0^{2\cos\theta}d\theta$　$0\le r\le2\cos\theta$. $-\frac{\pi}{2}\le\theta\le\frac{\pi}{2}$.

$=-\int_0^{2\pi}\frac16(4-4\cos^2\theta)^{\frac32}-\frac16\times4^{\frac32}d\theta$

$=-\frac16\int_0^{2\pi}(2\sin\theta)^3-2^3d\theta$

$=\frac43\int_0^{2\pi}1-\sin^3\theta\,d\theta=\frac43(2\pi-\int_0^{2\pi}\sin^3\theta)d\theta$

$=\frac43(2\pi-\frac{2}{3\times1})=\cancel{\frac43(\pi-\frac23)}$　$\frac43\pi-\frac{16}{9}$

(2) $\iint_Dxy\,dxdy$. $D:x^4+y^2\le1$. $x\ge0$, $y\ge0$.

$x^2=r\cos\theta$, $y=r\sin\theta$.　$x\ge0$, $x=\sqrt{r\cos\theta}$.

$\begin{vmatrix}\sqrt{\cos\theta}\frac{1}{2\sqrt r}&-\sqrt r\frac{1}{2\sqrt{\cos\theta}}\sin\theta\\\sin\theta&r\cos\theta\end{vmatrix}$

$=\frac{\sqrt r}{2\sqrt{\cos\theta}}\sin^2\theta+\frac{r}{2\sqrt r}\cos\theta\sqrt{\cos\theta}=\frac{\sqrt r}{2}(\cos\theta\sqrt{\cos\theta}+\frac{\sin^2\theta}{\sqrt{\cos\theta}})$

$=\frac{\sqrt r}{2}\frac{1}{\sqrt{\cos\theta}}(\cancel{\cos^2\theta+\sin^2\theta})$

$\int_0^1\int_0^{\frac{\pi}{2}}\sqrt{r\cos\theta}\,r\sin\theta\frac{\sqrt r}{2\sqrt{\cos\theta}}(\cos^2\theta-\sin^2\theta)dr\,d\theta$

$=\int_0^1\int_0^{\frac{\pi}{2}}\frac{r^2}{2}\sin\theta(\cancel{\cos^2\theta}-\sin^2\theta)dr\,d\theta$　$=\int_0^1\int_0^{\frac{\pi}{2}}\frac{r^2}{2}\sin\theta\,dr\,d\theta$

$=\frac12\int_0^1r^2dr\int_0^{\frac{\pi}{2}}\sin\theta\cancel{\cos^2\theta}-\sin^3\theta\,d\theta$　$=\frac16r^3\times1=\frac16$

$=\frac16r^3|_0^1\times(\int_0^{\frac{\pi}{2}}\cancel{\cos^2\theta\,d\cos\theta}-\frac{2}{3\times1})$

$=\frac16\times(-\frac13\cos^3\theta|_0^{\frac{\pi}{2}}-\frac23)=\frac16(-\frac13)=$

(3) $\iint_D\frac{|xy|(x^{\frac23}+1)}{x^{\frac23}+y^{\frac23}+2}dxdy$. $x^4+y^4\le a^2$.

$x=\sqrt{r\cos\theta}$, $y=\sqrt{r\sin\theta}$.　$r^2\le a^2$. $-a\le r\le a$.

$=(r\cos\theta)^{\frac12}$　$=(r\sin\theta)^{\frac12}$.　$\cos\theta$

$\begin{vmatrix}\sqrt{\cos\theta}\frac{1}{2\sqrt r}&-\sqrt r\frac{1}{2\sqrt{\cos\theta}}\sin\theta\\\sqrt{\sin\theta}\frac{1}{2\sqrt r}&\sqrt r\frac{1}{2\sqrt{\sin\theta}}\cos\theta\end{vmatrix}=\frac14\frac{\sqrt{\cos\theta}}{\sqrt{\sin\theta}}\cos\theta+\frac14\frac{\sqrt{\sin\theta}}{\sqrt{\cos\theta}}\sin\theta$

$=\frac14\frac{\cos^2\theta+\sin^2\theta}{\sqrt{\sin\theta}\sqrt{\cos\theta}}=\frac{1}{4\sqrt{\cos\theta\sin\theta}}$

$4\int_0^a\int_0^{\frac{\pi}{2}}\frac{r\sqrt{\cos\theta\sin\theta}((r\cos\theta)^{\frac13}+1)}{(r\cos\theta)^{\frac13}+(r\sin\theta)^{\frac13}+2}\times\frac{1}{4\sqrt{\cos\theta\sin\theta}}dr\,d\theta.$

$=\cancel{4}\int_0^a\int_0^{\frac{\pi}{2}}\frac{r(r^{\frac13}\cos\theta^{\frac13}+1)}{(r\cos\theta)^{\frac13}+(r\sin\theta)^{\frac13}+2}dr\,d\theta.$

积分区域关于$y=x$对称.

$\iint_D\frac{|xy|(x^{\frac23}+1)}{x^{\frac23}+y^{\frac23}+2}dxdy=\iint_D\frac{|xy|(y^{\frac23}+1)}{x^{\frac23}+y^{\frac23}+2}dxdy.$

$=\frac12\iint_D[\frac{|xy|(x^{\frac23}+1)}{x^{\frac23}+y^{\frac23}+2}+\frac{|xy|(y^{\frac23}+1)}{x^{\frac23}+y^{\frac23}+2}]dxdy.$

$=\frac12\iint_D|xy|dxdy.$

$=\frac{2}{\cancel{4}}\int_0^a\int_0^{\frac{\pi}{2}}r\sqrt{\cos\theta\sin\theta}\times\frac{1}{4\sqrt{\cos\theta\sin\theta}}d\theta\,dr=\frac12r^2|_0^a\times\frac{\pi}{2}=\frac{\pi}{8}a^2$

(4) $\iint_D\arctan|\frac{x-y}{x+y}|dxdy$. $D:\frac{x^2}{a^2}+\frac{y^2}{b^2}\le1$.

$\arctan|\frac{x-y}{x+y}|+\arctan|\frac{x+y}{x-y}|=\frac{\pi}{2}.$

$\frac12\iint_D(\arctan|\frac{x-y}{x+y}|+\arctan|\frac{x+y}{x-y}|)dxdy=\frac{\pi^2}{4}ab.$

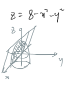
(5) $\iint_De^{\frac{x-y}{x+y}}dxdy$

$u=x-y$　$x=\frac{u+v}{2}$　$(0,1)\to(-1,1)$

$v=x+y$.　$y=\frac{v-u}{2}$.　$(1,0)\to(1,1)$

$\int_0^1\int_{-u}^ue^{\frac{u}{v}}dv\,du.$

69. $z=8-x^2-y^2$. $z=2y$.



交线 $8-x^2-y^2=2y$, $x^2+(y+1)^2=9$.

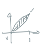
投影 $x^2+(y+1)^2\le9$.

$\iint_D(8-x^2-y^2-2y)dxdy=\iint_D(9-x^2-(y+1)^2)dxdy.$

71. $f(x)$在$[0,1]$连续

$\int_0^1dx\int_{x^2}^{\sqrt x}f(y)dy=\int_0^1(\sqrt x-x^2)f(x)dx.$

$\int_0^1dx\int_{x^2}^{\sqrt x}f(y)dy=\int_0^1dy\int_{y^2}^{\sqrt y}f(y)dx$



$=\int_0^1f(y)(\sqrt y-y^2)dy.$

72

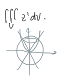
三重.

78.(1) $\int_0^1dx\int_x^1dy\int_y^1y\sqrt{1+z^4}dz$

79.(1) $\iiint_Vxy^2z^3dV$. $z=xy$, $z=0$, $y=x$, $x=1$.

$\int_0^1\int_0^x\int_0^{xy}xy^2z^3dV.$

80.(1) $\iiint_Vz^2dV$. $x^2+y^2+z^2=2$, $z=x^2+y^2$. $(0,0,1)$.



$\iint_D\int_{x^2+y^2}^{\sqrt{2-x^2-y^2}}z^2dz\,ds.$

$z^2+z-2=0$

$(z+2)(z-1)=0$

$z=1$.

$x=r\cos\theta$. $y=r\sin\theta$.

$\int_0^1dr\int_0^{2\pi}d\theta\int_{r^2}^{\sqrt{2-r^2}}z^2dz$

$=2\pi\int_0^1\int_{r^2}^{\sqrt{2-r^2}}z^2r\,dz\,dr$

$=\pi\int_0^1\frac13[(\sqrt{2-r^2})^3-(r^2)^3]dr^2$

$=\frac{\pi}{3}\int_0^1(\sqrt{2-t})^3-t^3dt$

$=\frac{\pi}{3}[\int_0^1(2-t)^{\frac32}dt-\int_0^1t^3dt]$

$=\frac{\pi}{3}(-\frac25(2-t)^{\frac52}|_0^1-\frac14t^4|_0^1)$

$\frac25+\frac14=\frac{8+5}{20}$

$=\frac{\pi}{3}(-\frac25(1-2^{\frac52})-\frac14)=\frac{\pi}{3}(\frac25\times2^{\frac52}-\frac{13}{20})$

$=(\frac{8}{15}\sqrt2-\frac{13}{60})\pi.$

81.(1) $\iiint_V(\sqrt{x^2+y^2+z^2}+x-y^3)dV$. $x^2+y^2+z^2=2z$.

奇　奇　对称.

$\iiint_VxdV=0$, $\iiint_Vy^3dV=0$

83.(1) $\iiint_V(x-1)^3dV$. $(x-1)^2+(y-2)^2+(z-3)^2\le1$.