MIT行列式随记

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矩阵可逆 $\Longleftrightarrow$ 行列式非零.

① $\det(I)=1$

② exchange rows, reverse sign

$\det(P)$ 置换矩阵. even 1. odd −1

③ $\begin{vmatrix} ta & tb \\ c & d \end{vmatrix} = t \begin{vmatrix} a & b \\ c & d \end{vmatrix}$ （n−1 行不动）

$\begin{vmatrix} a+a' & b+b' \\ c & d \end{vmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} + \begin{vmatrix} a' & b' \\ c & d \end{vmatrix}$.

第一行 线性函数

④ 2 equal rows of A, $\det(A)=0$

证明：Exchange rows $\begin{cases} \text{same matrix} \\ \text{reverse sign} \end{cases} \Rightarrow 0$.

⑤ substract $l \times$ row i from row k.

det doesn't change

$\begin{vmatrix} a & b \\ c-la & d-lb \end{vmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} + \begin{vmatrix} a & b \\ -la & -lb \end{vmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} - l \begin{vmatrix} a & b \\ a & b \end{vmatrix}$.

⑥ Row of zeros $\rightarrow \det A = 0$

证明：③ 令 $t=0$

⑦ $U = \begin{vmatrix} d_1 & * & * & * \\ 0 & d_2 & * & * \\ 0 & 0 & \ddots & \vdots \\ 0 & 0 & 0 & d_n \end{vmatrix} = d_1 \times d_2 \cdots d_n$ 　pivot 之积.

（都不为0）　若有0 → 行⑥ 行为0.

证明：⑤ 消元，③提因子，①可得. $d_1 d_2 \cdots d_n \begin{vmatrix} 1 & & \\ & \ddots & \\ & & 1 \end{vmatrix}$

⑧ $\det A = 0 \Longleftrightarrow A$ is singular.

$\det A \neq 0 \Longleftrightarrow A$ is invertible

$\begin{vmatrix} a & b \\ c & d \end{vmatrix} \begin{cases} a \neq 0. & \begin{vmatrix} a & b \\ 0 & d - \frac{cb}{a} \end{vmatrix} \\ a = 0. & \text{交换行，得出} \rightarrow 0 \end{cases}$

⑨ $\det(AB) = (\det A)(\det B)$

$\det A^{-1}$. $\det(A^{-1}A) = \det I = \det A^{-1} \cdot \det A = 1$

$\Rightarrow \det A^{-1} = (\det A)^{-1}$.

$\det A^2 = (\det A)(\det A)$　$\det \underset{\text{2行}}{2A} = 2^n \det A$

⑩ $\det A^T = \det A$

Prove. $|A^T| = |U^T L^T|$, $|A| = |LU|$

U 上三角，L 下三角

初等 → 三角阵 → 对角阵. （⑤⑦）

$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = \begin{vmatrix} a & 0 \\ c & d \end{vmatrix} + \begin{vmatrix} 0 & b \\ c & d \end{vmatrix}$

$= \underline{\begin{vmatrix} a & 0 \\ c & 0 \end{vmatrix}}_{0} + \begin{vmatrix} a & 0 \\ 0 & d \end{vmatrix} + \begin{vmatrix} 0 & b \\ c & 0 \end{vmatrix} + \underline{\begin{vmatrix} 0 & b \\ 0 & d \end{vmatrix}}_{0}$

$= \begin{vmatrix} a & 0 \\ 0 & d \end{vmatrix} - \begin{vmatrix} c & 0 \\ 0 & b \end{vmatrix}$

$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & 0 & 0 \\ & & \\ & & \end{vmatrix} + \begin{vmatrix} 0 & a_2 & 0 \\ & & \\ & & \end{vmatrix} + \begin{vmatrix} 0 & 0 & a_3 \\ & & \\ & & \end{vmatrix}$

survivors ($\neq 0$) $= |b_1\ 0\ 0|\ |0\ b_2\ 0|\ |0\ 0\ b_3|$

each row & column has a number $= \underbrace{\begin{vmatrix} a_1 & 0 & 0 \\ 0 & b_2 & 0 \\ 0 & 0 & c_3 \end{vmatrix}}_{a_1 b_2 c_3} + \begin{vmatrix} 0 & a_2 & 0 \\ b_1 & 0 & 0 \\ 0 & 0 & c_3 \end{vmatrix} + \begin{vmatrix} a_1 & 0 & 0 \\ 0 & 0 & b_3 \\ 0 & c_2 & 0 \end{vmatrix}$

交换次数的奇偶数决定正负　　$-a_2 b_1 c_3$　　$-a_1 b_3 c_2$

$a_2 b_3 c_1$

$a_3 b_1 c_2 - a_3 b_2 c_1$

$\det A = \sum_{n!\ \text{terms}} \pm a_{1\alpha} a_{2\beta} a_{3\gamma} \cdots a_{n\omega}$　展开

$(\alpha, \beta, \gamma \ldots \omega) = \text{perm of } (1, 2, \ldots n)$

第1行 n 种

第2行 n−1 种

Cofactors. $\det = a_1 \underbrace{(b_2 c_3 - b_3 c_2)}_{\text{factor}}$ → n−1 阶小矩阵　co: going with a_1

$- a_2 \underbrace{(b_1 c_3 - b_3 c_1)}$ → n−1 阶小矩阵

$+ a_3 (b_1 c_2 - b_2 c_1)$.

Cofactor of a_{ij}.

$\det(\begin{matrix} n-1 \text{ matrix} \\ \text{without } r_i \,\&\, c_j \end{matrix})$　sign: $i+j$ even/odd.

$\det(A) = a_{11} C_{11} + a_{12} C_{12} + \cdots + a_{1n} C_{1n}$ (along r_1).

代数余子式

可按行/列展开

eg. $A_4 = \begin{vmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{vmatrix}$

$A_1 = |1| \rightarrow \det |A_1| = 1$.

$A_2 = \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} \rightarrow \det |A_2| = 0$

$A_3 = \begin{vmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{vmatrix} - \begin{vmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{vmatrix} \rightarrow \det |A_3| = -1$

$A_4 = 1 \times |A_3| - 1 \times |A_2|$ 　按列展开 除1其他为0

$\Rightarrow A_n = |A_{n-1}| - |A_{n-2}|$