补充题题题题题

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$$\begin{vmatrix} a & a+d & \cdots & a+(n-2)d & a+(n-1)d \\ a+d & a+2d & \cdots & a+(n-1)d & a \\ \vdots & \vdots & & & a+d \\ & & & & \vdots \\ a+(n-2)d & a+(n-1)d & a & \cdots & a+(n-3)d \\ a+(n-1)d & a & a+d & \cdots & a+(n-2)d \end{vmatrix}$$

$$= \begin{vmatrix} na+\frac{(0+(n-1))n}{2}d \\ na+\frac{n(n-1)}{2}d \\ \vdots \\ na+\frac{n(n-1)}{2} \end{vmatrix} \Rightarrow \left(na+\frac{n(n-1)}{2}\right)d \begin{vmatrix} 1 & a+d & \cdots & a+(n-1)d \\ 1 & a+2d & \cdots & a \\ \vdots & \vdots & & \vdots \\ 1 & a+(n-1)d & \cdots & a+(n-3)d \\ 1 & a & \cdots & a+(n-2)d \end{vmatrix}$$

$$= \left[na+\frac{n(n-1)d}{2}\right] \begin{vmatrix} 1 & a+d & \cdots & a+(n-1)d \\ 0 & d & \cdots & -(n-1)d \\ 0 & d & \cdots & d \\ \vdots & \vdots & & \vdots \\ 0 & d & & d \\ 0 & -(n-1)d & \cdots & d \end{vmatrix}$$

$$= \left[na+\frac{n(n-1)d}{2}\right] d^{n-1} \begin{vmatrix} 1 & a+d & \cdots & a+(n-1)d \\ 0 & 1 & \cdots & 1-n \\ 0 & 1 & & 1 \\ \vdots & \vdots & & \vdots \\ 0 & 1 & & 1 \\ 0 & 1-n & \cdots & 1 \end{vmatrix}$$

$$= \left[na+\frac{n(n-1)d}{2}\right] d^{n-1} \begin{vmatrix} 1 & 1 & 1 & \cdots & 1 & 1-n \\ 1 & 1 & 1 & \cdots & 1-n & 1 \\ 1 & 1 & 1 & & 1 & 1 \\ \vdots & & & & & \vdots \\ 1-n & 1 & 1 & \cdots & & 1 \end{vmatrix}_{(n-1)}$$

$$= \left[na+\frac{n(n-1)d}{2}\right] d^{n-1} \begin{vmatrix} n-2+1-n & 1 & 1 & \cdots & 1 & 1-n \\ -1 & 1 & 1 & \cdots & 1-n & 1 \\ -1 & 1 & 1 & & 1 & 1 \\ \vdots & & & & & \vdots \\ -1 & 1-n & 1 & & & 1 \\ -1 & 1 & 1 & \cdots & 1 & 1 \end{vmatrix}$$

$$= \left[na+\frac{n(n-1)}{2}d\right] d^{n-1} \begin{vmatrix} 1 & 1 & 1 & \cdots & 1 & 1-n \\ 0 & 0 & 0 & \cdots & -n & n \\ 0 & 0 & 0 & \cdots & n & 0 \\ \vdots & & & & & \vdots \\ 0 & -n & n & & 0 & 0 \\ 0 & n & 0 & \cdots & 0 & 0 \end{vmatrix}$$

$$= \left[na+\frac{n(n-1)}{2}d\right] d^{n-1} (-1)^{\frac{(n-1)(n-2)}{2}} n^{n-2}$$

$$\Delta_n = \begin{vmatrix} c_1 & a_2 & a_3 & \cdots & a_n \\ a_1 & c_2 & a_3 & \cdots & a_n \\ a_1 & a_2 & c_3 & & \vdots \\ \vdots & & & \ddots & a_n \\ a_1 & a_2 & \cdots & a_{n-1} & c_n \end{vmatrix}$$

$$= \begin{vmatrix} 1 & -a_1 & -a_2 & -a_3 & \cdots & -a_n \\ 0 & c_1 & a_2 & a_3 & \cdots & a_n \\ 0 & a_1 & c_2 & a_3 & \cdots & a_n \\ 0 & a_1 & a_2 & c_3 & & \vdots \\ \vdots & \vdots & & & \ddots & a_n \\ 0 & a_1 & a_2 & \cdots & a_{n-1} & c_n \end{vmatrix}$$

$$= \begin{vmatrix} 1 & -a_1 & -a_2 & -a_3 & \cdots & -a_n \\ 1 & c_1-a_1 & 0 & 0 & \cdots & 0 \\ 1 & 0 & c_2-a_2 & 0 & \cdots & 0 \\ \vdots & \vdots & & \ddots & & \vdots \\ 1 & 0 & 0 & 0 & c_{n-1}-a_{n-1} & 0 \\ 1 & 0 & 0 & 0 & 0 & c_n-a_n \end{vmatrix}$$

Binet − Cauchy.　$A_{m\times n}, B_{n\times m}$

$$\det AB = \begin{cases} 0 & m>n \\ \det A \det B & m=n \\ \sum |A_m||B_m| & m<n \end{cases}$$

$$\begin{array}{c} \\ m \\ n \end{array} \begin{vmatrix} A & 0 \\ 0 & B \end{vmatrix} \Rightarrow \begin{vmatrix} A & 0 \\ I_n & B \end{vmatrix} = \begin{vmatrix} 0 & -AB \\ I_n & B \end{vmatrix}$$

$$= (-1)^{nm} |I_n| |-AB|$$

$$= (-1)^{n+mn} |AB|$$

$$|AB| = (-1)^{n+mn} \begin{vmatrix} A & 0 \\ I_n & B \end{vmatrix}$$

$$= (-1)^{n+mn} \sum A\begin{pmatrix} 1 & \cdots & s \\ v_1 & \cdots & v_s \end{pmatrix} (-1)^{s+v_s} (-1)^{n-s} \cdots \sum B\begin{pmatrix} v_1 & \cdots & v_s \\ 1 & \cdots & s \end{pmatrix}$$

$$= \sum A_m B_m$$

若 $A = \begin{pmatrix} a_1 & \cdots & a_n \\ b_1 & \cdots & b_n \end{pmatrix}$　$B = \begin{pmatrix} c_1 & d_1 \\ \vdots & \vdots \\ c_n & d_n \end{pmatrix}$

$$|AB| = \begin{vmatrix} \sum a_i c_i & \sum a_i d_i \\ \sum b_i c_i & \sum b_i d_i \end{vmatrix}_{2\times 2}$$

$$= \sum a_i c_i \sum b_i d_i - \sum a_i d_i \sum b_i c_i$$

$$= \sum (a_i b_j - a_j b_i)(c_i d_j - c_j d_i)$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & \\ & -1 \end{pmatrix} \begin{pmatrix} -1 & \\ & 1 \end{pmatrix}$$

$$(x \quad y \quad z \quad 1) \begin{pmatrix} A_1 & A_2 \\ B_1 & B_2 \\ C_1 & C_2 \\ D_1 & D_2 \end{pmatrix} = 0$$

$$\begin{matrix} x_1 & a_2 & a_3 & \cdots & a_n \\ a_1 & x_2 & a_3 & \cdots & a_n \\ a_1 & a_2 & x_3 & \cdots & a_n \\ \vdots & & & \ddots & \vdots \\ a_1 & a_2 & a_3 & \cdots & x_n \end{matrix} \rightarrow \begin{vmatrix} 1 & a_1 & a_2 & a_3 & \cdots & a_n \\ 0 & x_1 & a_2 & a_3 & \cdots & a_n \\ 0 & a_1 & x_2 & a_3 & \cdots & a_n \\ 0 & a_1 & a_2 & x_3 & & a_n \\ \vdots & & & & \ddots & \vdots \\ 0 & a_1 & a_2 & a_3 & & a_n \\ 0 & a_1 & a_2 & a_3 & \cdots & x_n \end{vmatrix}$$

$$= \begin{vmatrix} 1 & a_1 & a_2 & a_3 & \cdots & a_n \\ -1 & x_1-a_1 & 0 & 0 & & 0 \\ -1 & 0 & x_2-a_2 & 0 & & 0 \\ -1 & 0 & 0 & x_3-a_3 & & 0 \\ \vdots & & & & \ddots & \vdots \\ -1 & 0 & 0 & 0 & \cdots & x_n-a_n \end{vmatrix}$$

$x_i \neq a_i$,

$$\begin{vmatrix} 1-\sum\frac{a_i}{x_i-a_i} & a_1 & a_2 & \cdots & a_n \\ 0 & x_1-a_1 & & & \\ 0 & & x_2-a_2 & & \\ \vdots & & & \ddots & \\ 0 & & & & x_n-a_n \end{vmatrix}$$

$$= \left(1-\sum\frac{a_i}{x_i-a_i}\right)\prod(x_i-a_i)$$