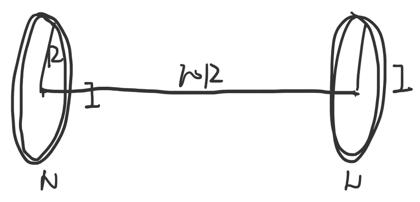
能摸到的期末往年题

2022年6月23日 星期四　　下午5:42

2018 A卷

1.



(1) 一个线圈轴线上的 B.

(2) 磁场强度的区元图.

(3) 证明 a位于某处. (a为线圈半径).

(1) $dB = \frac{\mu_0}{4\pi}\frac{IRd\theta}{R^2+z^2}$

$B = \int_0^{2\pi} \frac{\mu_0}{4\pi}\frac{IR}{R^2+z^2}\frac{R}{\sqrt{R^2+z^2}}d\theta = \frac{\mu_0 IR^2}{2(R^2+z^2)^{3/2}}$

(2) $\delta\theta_c = \sqrt{\frac{B_{min}}{B_{max}}}$

$B_{min} = 2\times\frac{\mu_0 IR^2}{2(R^2+(5R)^2)^{3/2}} = \frac{2\mu_0 IR^2}{2\cdot 126 R^3}$

$B_{max} = \frac{\mu_0 IR^2}{2(R^2)^{3/2}} + \frac{\mu_0 IR^2}{2(R^2+(10R)^2)^{3/2}}$

$\delta\theta = \sqrt{\frac{B_{min}}{B_{max}}} = \sqrt{\frac{\frac{2}{(26R^2)^{3/2}}}{\frac{1}{(R^2)^{3/2}}+\frac{1}{(R^2+100R^2)^{3/2}}}} = \sqrt{\frac{\frac{2}{(26)^{3/2}}}{1+\frac{1}{(101)^{3/2}}}} = 0.12276$

$\theta \approx 7.05°$

(3) $a = \frac{mv_\perp}{qB}$, $a^2 = \frac{m^2v_\perp^2}{q^2B^2} = \frac{2m\mu}{q^2B}$

$\frac{1}{2}mv_\parallel^2 + \frac{1}{2}mv_\perp^2 = const.$

$\mu = \frac{mv_\perp^2}{2B} = const.$

$a^2B = \frac{2m\mu}{q^2}$

$a\sqrt{B} = \sqrt{\frac{2m\mu}{q^2}} = const.$

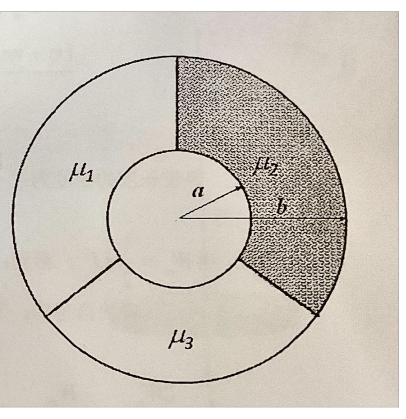
2.（16 分）同轴电缆

同轴电缆的内导体是半径为 a 的空心圆柱，外导体是半径为 b 的薄圆柱面，其厚度可以忽略不计，内、外导体间填充有绝对磁导率分别为μ_1、μ_2和μ_3的三种磁介质，每种磁介质均占三分之一的圆柱体积，分界面正好沿半径方向，如图所示。设内圆柱面内沿轴向流有大小相等、方向相反的电流，电流面密度为 i；求：

（1）各区域的磁感应强度和磁场强度；（8 分）

（2）同轴电缆单位长度所储存的磁能量；（4 分）

（3）同轴电缆单位长度的自感。（4 分）



(1) $r<a$, $B=0$. $H=0$

$a<r<b$, $\frac{B}{\mu_1}\frac{2\pi r}{3} + \frac{B}{\mu_2}\frac{2\pi r}{3} + \frac{B}{\mu_3}\frac{2\pi r}{3} = 2\pi a i$,

$\frac{B}{\mu_1}+\frac{B}{\mu_2}+\frac{B}{\mu_3} = \frac{3ai}{r}$, $B = \frac{3ai}{r(\frac{1}{\mu_1}+\frac{1}{\mu_2}+\frac{1}{\mu_3})}$

$r>b$, $B=0$, $H=0$.

$\therefore \vec{B} = \begin{cases} 0 & r<a \\ \frac{3ai}{r(\frac{1}{\mu_1}+\frac{1}{\mu_2}+\frac{1}{\mu_3})}\vec{e_\theta} & a<r<b \\ 0 & r>b \end{cases}$

$\vec{H} = \begin{cases} 0 & r<a \\ \frac{3\mu_i ai}{r(\frac{1}{\mu_1}+\frac{1}{\mu_2}+\frac{1}{\mu_3})}\vec{e_\theta} & a<r<b \\ 0 & r>b \end{cases}$

(2) $w = \frac{1}{2}\vec{B}\cdot\vec{H} = \frac{1}{2}\frac{9a^2i^2\mu_i}{r^2(\frac{1}{\mu_1}+\frac{1}{\mu_2}+\frac{1}{\mu_3})^2}$

$W = \iiint w dV = \frac{9a^2i^2}{2(\frac{1}{\mu_1}+\frac{1}{\mu_2}+\frac{1}{\mu_3})^2}\iiint_V \frac{\mu_i}{r^2}dV$

$= \int_a^b\int_0^l(\int_0^{\frac{2\pi}{3}}\frac{\mu_1}{r^2}rd\theta + \int_{\frac{2\pi}{3}}^{\frac{4\pi}{3}}\frac{\mu_2}{r^2}rd\theta + \int_{\frac{4\pi}{3}}^{2\pi}\frac{\mu_3}{r^2}rd\theta)\,dl\,dr$

$= \int_a^b\frac{1}{r}dr\cdot l\cdot(\int_0^{\frac{2\pi}{3}}\mu_1 d\theta + \int_{\frac{2\pi}{3}}^{\frac{4\pi}{3}}\mu_2 d\theta + \int_{\frac{4\pi}{3}}^{2\pi}\mu_3 d\theta)$

$= \frac{9a^2i^2\,l\,\frac{2\pi}{3}}{2(\frac{1}{\mu_1}+\frac{1}{\mu_2}+\frac{1}{\mu_3})^2}(\mu_1+\mu_2+\mu_3)\ln\frac{b}{a}$.

(3) $L_i = \frac{2W}{I^2 l} = \frac{2W}{2\pi a i\,l} = \frac{\frac{9a^2i^2}{(\frac{1}{\mu_1}+\frac{1}{\mu_2}+\frac{1}{\mu_3})^2}(\mu_1+\mu_2+\mu_3)\ln\frac{b}{a}}{\pi a i}$

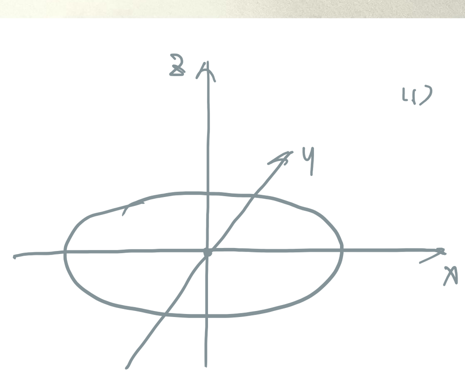
$= \frac{3^2 a i(\mu_1+\mu_2+\mu_3)}{2\pi(\frac{1}{\mu_1}+\frac{1}{\mu_2}+\frac{1}{\mu_3})^2}\ln\frac{b}{a}$

（1）一个半径为 a，非常薄（厚度为 b）的导体圆盘放置在 xy 平面上，导体的电导率为 σ，磁导率为μ_0，原点在圆盘中心，空间加上磁场为：$\vec{B} = B_0\cos(\omega t+\varphi)\vec{e}_z$，请给出圆盘上半径为 r 处的涡流密度 j_r （6 分）

（2）请求出圆盘的总磁矩，并给出远处 P 点（$r \gg a$）由涡流产生的磁感应强度。（6 分）

（3）导体置于随时间变化的磁场中时，导体内部会出现"涡流"，即导体中自由电子在涡旋电场作用下形成的电流，涡旋电流又产生磁场，相当于一种"自散"效应，如果导体的电导率为σ_0，磁导率为μ_0，当涡流达到稳恒流动时（$\nabla\cdot\vec{j}_f = 0$），请证明：涡流密度 j_f 满足以下方程：（5 分）

$$\nabla^2\vec{j}_f = \sigma\mu_0\frac{\partial\vec{j}_f}{\partial t}$$



(1) $\oint\vec{E}\cdot d\vec{l} = -\iint\frac{d\vec{B}}{dt}\cdot d\vec{S}$

$E\times 2\pi r = -\frac{dB}{dt}\pi r^2 = \omega B_0\sin(\omega t+\varphi)\pi r^2$

$E = \frac{\omega B_0\sin(\omega t+\varphi)r}{2}$

$j_r = \sigma E = \frac{1}{2}\sigma\omega B r\sin(\omega t+\varphi)$

(2) $\vec{m} = I\cdot S = \int_0^a\vec{j_r}\cdot d\vec{S} = \frac{1}{2}\sigma\omega B\sin(\omega t+\varphi)\int_0^{2\pi}\int_0^a r^2 dr d\theta$

$= \frac{\pi a^3}{3}\sigma\omega B\sin(\omega t+\varphi)\vec{e_z}$

$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{l}}{r^2}$, $\vec{B} = -\frac{\mu_0\vec{m}}{4\pi r^3} + \frac{\mu_0\vec{r}(\vec{m}\cdot\vec{r})}{4\pi r^5}$

$dm = \pi r^2 dI = \pi r^2 j dr\, b$

$m = \int_0^a dm = \frac{1}{2}\sigma\omega B_0\pi b\sin(\omega t+\varphi)\int_0^a r^3 dr$

$= \frac{1}{8}\pi\sigma\omega B_0 b a^4\sin(\omega t+\varphi)$

$\begin{cases} B_r = \frac{\mu_0}{4\pi}\frac{2m}{r^3} \\ B_\theta = -\frac{\mu_0 m}{4\pi r^3} \end{cases}$

(3) $\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$,

$\nabla\cdot\vec{j} = 0$, $\vec{j} = \sigma\vec{E}$, $\nabla\times\vec{j} = -\sigma\frac{\partial\vec{B}}{\partial t}$

$\nabla\times(\nabla\times\vec{j_r}) = -\sigma\frac{\partial(\nabla\times\vec{B})}{\partial t} = -\sigma\frac{\partial\vec{j}}{\partial t}$

$\nabla^2\vec{j_r} = \sigma\frac{\partial\vec{j}}{\partial t}$

2006-2007

Maxwell

一、1. 2.

$\oiint_S\vec{D}\cdot d\vec{S} = \iiint_V\rho_q dV$ 　　$\nabla\cdot\vec{D} = \rho$ 　有源

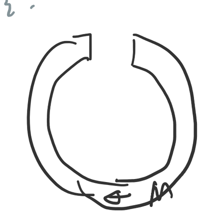
$\oint\vec{E}\cdot d\vec{l} = -\oiint_S\frac{\partial\vec{B}}{\partial t}dS$. 　　$\nabla\times\vec{E} = -\nabla\cdot\frac{\partial\vec{B}}{\partial t}$ 　磁→电

$\oiint_S\vec{B}\cdot d\vec{S} = 0$ 　　$\nabla\cdot\vec{B} = 0$ 　无源

$\oint\vec{H}\cdot d\vec{l} = \oiint_S i + \frac{\partial\vec{D}}{\partial t}dS$ 　　$\nabla\times\vec{H} = \nabla\cdot(\frac{\partial\vec{D}}{\partial t}+I)$. 电→磁

双所得量.

3.



$\frac{\vec{B}}{\mu_0} = \vec{H} - \vec{M}_1 =$

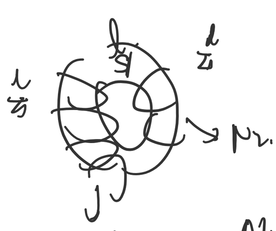
$\frac{\vec{B}}{\mu} = \vec{H}$ 　$\frac{\mu}{\mu_0}\vec{H} = \vec{H} - \vec{M}$.

$\vec{M} = (1-\frac{\mu}{\mu_0})\vec{H}$.

Maxwell.

4.

5.



N_1, I. 　$N_1 I = \Phi\frac{l}{\mu S}$, $\Phi = \frac{\mu S N_1 I}{l}$. 同路中.

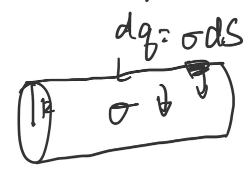
(1) 对N_1的 $\psi = \frac{N_1^2\mu S I}{l}$, $L = \frac{N_1^2\mu S}{l}$.

同理: $L_2 = \frac{N_2^2\mu S}{l}$.

(2) 对N_2的 $\psi = N_2\Phi = \frac{N_1N_2\mu S I}{l}$

$M = \frac{\psi}{I} = \frac{N_1N_2\mu S}{l}$.

二、1.



$dq = \sigma dS$ 　角加速度 β. 　角速度 $w = \beta t$.

(1) $dq = \sigma dS = \sigma R d\theta dl$. 　$I = \frac{dq}{dt} = \frac{\sigma dS}{dt} = \frac{\sigma R d\theta dl}{dt} = 2\pi\sigma R\beta t dl$

$T = \frac{2\pi}{w} = \frac{2\pi}{\beta t}$, $dI = \frac{dq}{T} = \frac{\sigma R d\theta dl}{\frac{2\pi}{\beta t}} = \frac{\sigma R\beta t d\theta dl}{2\pi}$

$I = \sigma R\beta t L$. 　$I = 2\pi R l\sigma w$. 　$i = 2\pi R\sigma w$

σ. $r>R$, $E\times 2\pi r L = \varepsilon_0\sigma R\times 2\pi L$, $E = \frac{R}{r}\sigma\varepsilon_0$.

$D = \frac{R}{r}\sigma$.

$B = \int\frac{4\pi}{\mu_0}\frac{Idl}{R^2} = \frac{4\pi}{\mu_0}\int_0^{2\pi}\frac{IRd\theta}{R^2}$ 　$Bdl = \mu_0 I = \mu_0 i\,dl$

$= \frac{4\pi}{\mu_0}\frac{\sigma R\beta t L R 2\pi}{R^2} = \frac{8\pi^2\sigma\beta t L}{\mu_0}$. 　$B = 2\pi R\mu_0\sigma\beta t\hat{l}$

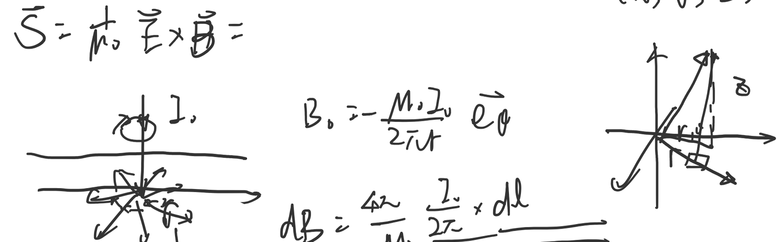
(2) $\oint_l\vec{E}\cdot d\vec{l} = -\iint_S\frac{\partial\vec{B}}{\partial t}\cdot d\vec{S}$ 　$\oint\vec{E}\cdot dl = -\iint_S\frac{\partial\vec{B}}{\partial t}d\vec{S}$. $E = \mu_0\sigma\beta 2\pi^2 l\times\rho$.

$\vec{E}\times 2\pi R = -\frac{8\pi^2\sigma\beta L}{\mu_0}\times\pi R^2$.

$\vec{E} = -\frac{4\pi^2\sigma\beta L}{\mu_0}R\vec{e_\theta}$ 　$(r\cos\theta, r\sin\theta, 0)$ $(r_0\cos\theta, r_0\sin\theta, z)$ (r_0, θ, z).

(3) $\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B} =$

2.



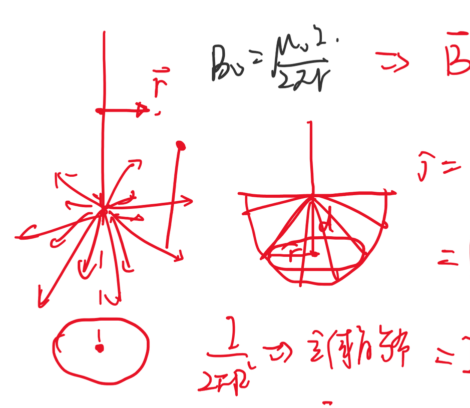
$B_0 = -\frac{\mu_0 I_0}{2\pi r}\vec{e_\theta}$

$dB = \frac{4\pi}{\mu_0}\frac{\frac{I_0}{2\pi}\times dl}{\sqrt{(r\cos\theta - r_0\cos\theta_0)^2 + (r\sin\theta - r_0\sin\theta)}}$

$i = \frac{I_0}{2\pi}$.

$B = -\frac{\mu_0 I_0}{2\pi r}\vec{e_\theta}$. 　半空间内各个方向.

$dB = \frac{4\pi}{\mu_0}\frac{\frac{I}{2\pi}dl}{\sqrt{(l\cos\theta - r_0\cos\theta)^2 + (l\sin\theta - r_0\sin\theta)^2 + z^2}}$



$B_0 = \frac{\mu_0 I}{2\pi r}$ ⇒ $\vec{B} = \frac{\mu_0 I}{2\pi r^2}\vec{r}\times\vec{n}$.

$j = \iint\vec{j}\cdot d\vec{S}$

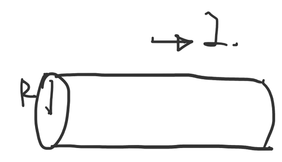
$= \int_0^{\pi}\int_0^{\frac{\pi}{2}}\frac{I}{2\pi(r^2-d^2)^{3/2}}(r^2-d^2)^{\frac{1}{2}}\sin\theta d\theta d\varphi$

$\frac{I}{2\pi r^2}$ ⇒ 立体角 $= I\int_0^{\theta}\sin\theta d\theta = I(1-\frac{d}{\sqrt{r^2+d^2}})$

$R^2\sin\theta$ 立元.

$\vec{B} = \frac{\mu_0 I}{2\pi r}(1-\frac{d}{\sqrt{r^2+d^2}})\vec{r}\times\vec{n}$.

3.



$I = nqvSe$

圆柱电子密度 n. e

e的电子分布.

$E\times 2\pi r\cdot l = \begin{cases}\frac{ne\pi r^2 l}{\varepsilon_0} & r<R \\ \frac{ne\pi R^2 l}{\varepsilon_0} & r>R\end{cases}$

$E = \begin{cases}\frac{ner}{2\varepsilon_0} & r<R \\ \frac{neR^2}{2r\varepsilon_0} & r>R\end{cases}$

$\Delta U = \frac{neR^2}{4\varepsilon_0}$

$H\times 2\pi r = \frac{I}{\pi R^2}\times\pi r^2$.

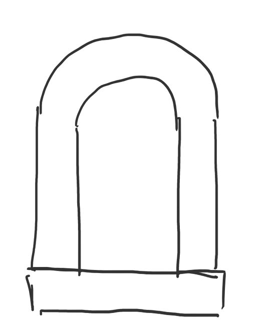
$B = \mu_0 H = \frac{\mu_0 I}{2\pi R^2}r$.

$Bev = Ee$.

$E = Bv = \frac{\mu_0 I^2 r}{2\pi^2 R^4 ne}$

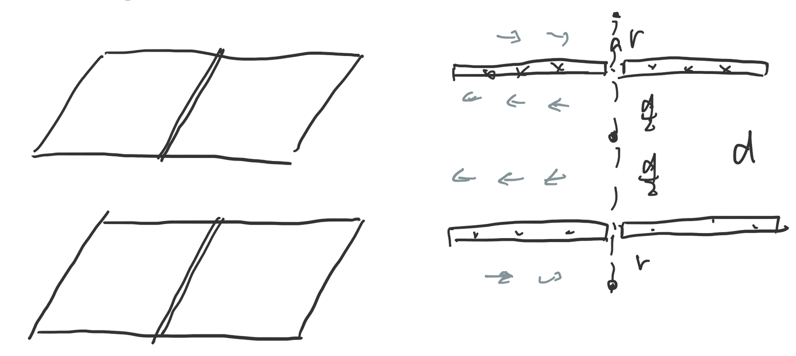
$U = \int Edl$

4.



2017 A卷

1.



$dB = \frac{\mu_0}{4\pi}\frac{Idl}{}$

$il\mu_0 = B\times 2l$

$B = \frac{\mu_0 i}{2}$.

II. $\mu_0 i$.

I. III. $\frac{\mu_0 i}{2}$.

$dB = \frac{\mu_0}{4\pi}\frac{idx}{x^2+r^2}\times\frac{r}{\sqrt{x^2+r^2}}$.

$B_{A1} = \int_{\frac{d}{2}}^{+\infty}\frac{\mu_0}{2\pi}\frac{ri}{(x^2+r^2)^{3/2}}dx = \frac{\mu_0 ri}{2\pi}\frac{x}{(x^2+r^2)^{\frac{1}{2}}}\Big|_{\frac{d}{2}}^{+\infty}$

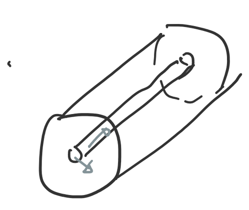
$= \frac{\mu_0 ri}{2\pi}\sqrt{\frac{1}{1+(\frac{r}{x})^2}}\Big|_{\frac{d}{2}}^{+\infty} = \frac{\mu_0 ri}{2\pi}(1-\sqrt{\frac{1}{1+(\frac{2r}{d})^2}})\vec{e}$

$B_A = \frac{\mu_0 ri}{2\pi}(1-\sqrt{\frac{1}{1+(\frac{2r}{d})^2}}) - \frac{\mu_0(d+r)i}{2\pi}(1-\sqrt{\frac{1}{1+(\frac{2r+2d}{d})^2}})\vec{e}$

B_C 同 B_A.

$B_B = \int_{\frac{d}{2}}^{+\infty}\frac{\mu_0}{\pi}\frac{\frac{d}{2}i}{(x^2+\frac{d^2}{4})^{3/2}}dx = \frac{\mu_0 di}{2\pi}(1-\sqrt{\frac{1}{1+(\frac{d}{d})^2}})$

2.



(1) $H\times 2\pi r = \begin{cases}\frac{\pi r^2}{\pi a^2}I & r<a \\ I & a<r<b \\ 0 & r>b\end{cases}$

$H = \begin{cases}\frac{r}{2\pi a^2}I \\ \frac{1}{2\pi r}I \\ 0\end{cases}$ 　$B = \begin{cases}\frac{\mu_r\mu_0 r}{2\pi a^2}I & r<a \\ \frac{\mu_r\mu_0}{2\pi r}I & a<r<b \\ 0 & r>b.\end{cases}$

$M = (\mu_r-1)H = \begin{cases}\frac{(\mu_r-1)r}{2\pi a^2}I & r<a \\ \frac{(\mu_r-1)}{2\pi r}I & a<r<b \\ 0 & r>b\end{cases}$

(2) $I(a) = \vec{n}_{12}\times(\vec{M}_2 - \vec{M}_1)$

$= \frac{(\mu_r-1)I}{2\pi}(\frac{r}{a^2}-\frac{1}{r})$ 　内.

$I(b) = \vec{n}_{12}(\vec{M}_3 - \vec{M}_2)$

$= -\vec{n}_{12}\times\vec{M}_2 = -\frac{(\mu_r-1)I}{2\pi r}$. 外.

(3) $w = \frac{1}{2}\vec{B}\cdot\vec{H} = \begin{cases}\frac{\mu_r\mu_0 r^2 I^2}{8\pi^2 a^4} & r<a \\ \frac{\mu_r\mu_0 I^2}{8\pi^2 r^2} & a<r<b\end{cases}$

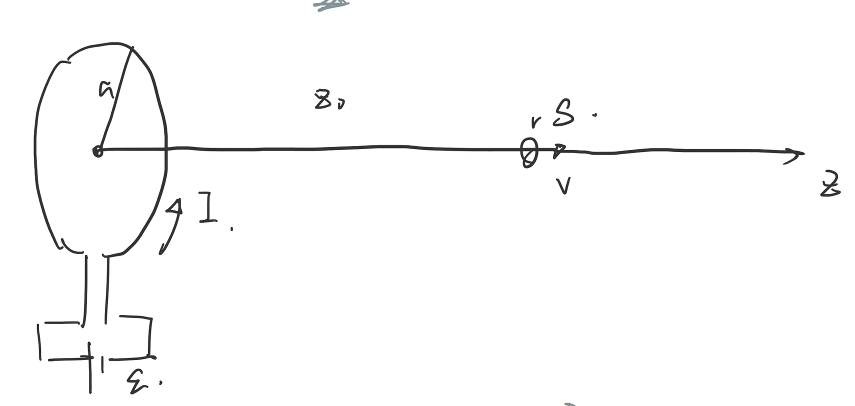
$W = \iiint_V w\,dV = 2\pi l[\int_0^a\frac{\mu_r\mu_0 I^2}{8\pi^2 a^4}r^2 + \int_a^b\frac{\mu_r\mu_0 I^2}{8\pi^2}\frac{1}{r^2}]r$

$= 2\pi l\times\frac{\mu_r\mu_0 I^2}{8\pi^2}(\frac{a^3}{3}\times\frac{1}{a^4} + (\frac{1}{a}-\frac{1}{b}))r$ 　柱坐标.

$= \frac{\mu_r\mu_0 l I^2}{4\pi}(\frac{4}{3a}-\frac{1}{b})$

$L = \frac{2W}{I^2 l} = \frac{\mu_r\mu_0}{2\pi}(\frac{4}{3a}-\frac{1}{b})$

3.



(1) $dB = \frac{\mu_0}{4\pi}\frac{Iad\theta}{a^2+z_0^2}\times\frac{a}{\sqrt{a^2+z_0^2}}$.

$B = \frac{\mu_0 I a^2}{2(a^2+z_0^2)^{\frac{3}{2}}}$ ✓

$\varepsilon = -\frac{\partial\Phi}{\partial t} = \frac{d(\frac{\mu_0 I a^2}{2(a^2+z_0^2)^{3/2}})S}{dt} = -\frac{\mu_0 I a^2 S}{2}\cdot(-\frac{3}{2}\frac{1}{(a^2+z_0^2)^{\frac{5}{2}}})2z_0 v$

$= \frac{3\mu_0 I a^2 S z_0 v}{2(a^2+z_0^2)^{\frac{5}{2}}}$

$I' = \frac{\varepsilon}{r} = \frac{3\mu_0 I a^2 S z_0 v}{2(a^2+z_0^2)^{\frac{5}{2}}r}$ ✓ 　z轴正方向看 逆. (正比为向逆)

(2) $\vec{F} = m\frac{\partial B}{\partial z}\vec{e_z}$

$= I'S\frac{\partial(\frac{\mu_0 I a^2}{2(a^2+z_0^2)^{3/2}})}{\partial z}\vec{e_z}$

$= \frac{3\mu_0 I a^2 S^2 z_0 v}{2(a^2+z_0^2)^{\frac{5}{2}}r}\times\frac{\mu_0 I a^2}{2(a^2+z_0^2)^{\frac{5}{2}}}\times(-\frac{3}{2})2z_0\vec{e_z}$

$= -\frac{9\mu_0^2 I^2 a^4 S^2 z_0^2 v}{4(a^2+z_0^2)^5 r}\vec{e_z}$ ✓

(3) 小线圈在此时的磁能.

$W = \vec{m}\cdot\vec{B} = I'S\frac{\mu_0 I a^2}{2(a^2+z_0^2)^{\frac{3}{2}}}$

$= \frac{3\mu_0^2 I^2 a^4 S^2 z_0 v}{4(a^2+z_0^2)^4 r}$

$\frac{dW}{dt} = \frac{3\mu_0^2 I^2 a^4 S^2 v}{4r}\frac{(a^2+z_0^2)^4 - z_0\cdot 4(a^2+z_0^2)^3 2z_0}{(a^2+z_0^2)^8}$

$= \frac{3\mu_0^2 I^2 a^4 S v}{4r}\frac{a^2 + 9z_0^2}{(a^2+z_0^2)^5}$

大线圈对外做功 $\Delta E = \frac{\frac{dW}{dt}}{I} = \frac{3\mu_0 I a^4 S}{4r}\frac{(a^2+9z_0^2)}{(a^2+z_0^2)^5}$

小线圈自感. 给大线圈的电动势 $\varepsilon_感$.

$\Delta E = -\varepsilon_感 = -(-\frac{d\Phi_{21}}{dt}) = M\frac{di}{dt}$.

$M = \frac{\Phi}{I} = \frac{\mu_0 a^2 S}{2(a^2+z_0^2)^{\frac{3}{2}}}$. 　$\frac{di}{dt} = \ldots$