期中样题

2022年5月25日 星期三　　下午4:07

一. 1. 2.05×10^{-9}　　2.18×10^{-18}　-13.6　　-5

$F=\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}$,　$W=\frac{1}{2}\varphi q\times 2=\varphi q=\frac{-e^2}{4\pi\varepsilon_0 r}$

2. $\frac{2}{3}b$. 对.

$E=\frac{Q}{4\pi r^2\varepsilon_0}$, $U=\int_a^b\frac{Q}{4\pi\varepsilon_0 r^2}dr=\frac{Q}{4\pi\varepsilon_0}(-\frac{1}{r})\Big|_a^b$

$=\frac{Q}{4\pi\varepsilon_0}(\frac{1}{a}-\frac{1}{b})$

$C=\frac{Q}{U}=\frac{4\pi\varepsilon_0}{\frac{1}{a}-\frac{1}{b}}$

$\Rightarrow Q=CU=\frac{4\pi\varepsilon_0 U}{\frac{1}{a}-\frac{1}{b}}$,

$\therefore E(a)=\frac{\frac{4\pi\varepsilon_0 U}{\frac{1}{a}-\frac{1}{b}}}{4\pi\varepsilon_0 a^2}=\frac{a^2U}{\frac{1}{a}-\frac{1}{b}}=\frac{a^3bU}{b-a}$　对.

$E'(a)=bU\times\frac{3a^2(b-a)+a^3}{(b-a)^2}=bU\times\frac{-2a^3+3a^2b}{(b-a)^2}=\frac{a^2bU(-2a+3b)}{(b-a)^2}$

$2a=3b$, $a=\frac{3}{2}b$

$E=\frac{27}{8}b^4U$ 不1

$E=\frac{Q}{4\pi r^2\varepsilon_0}$ $r\in[a,b]$ $=\frac{\frac{4\pi\varepsilon_0 U}{\frac{1}{a}-\frac{1}{b}}}{4\pi\varepsilon_0 r^2}=\frac{U}{(\frac{1}{a}-\frac{1}{b})r^2}$

$U=\frac{Q}{4\pi\varepsilon_0}(\frac{1}{a}-\frac{1}{b})$

$C=\frac{Q}{U}=\frac{4\pi\varepsilon_0}{\frac{1}{a}-\frac{1}{b}}$

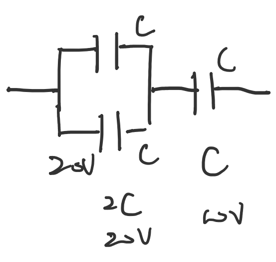
$Q=CU=\frac{4\pi\varepsilon_0}{\frac{1}{a}-\frac{1}{b}}U$.

$E(a)=\frac{U\,ab}{(b-a)a^2}=\frac{Ub}{(b-a)a}$

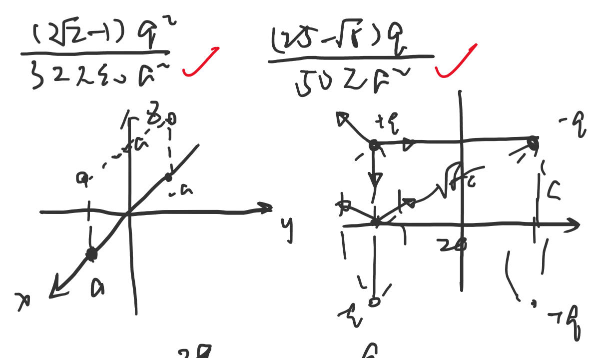
$E'(a)=Ub\times\frac{-(b-2a)}{(b-a)^2a^2}$　$\Rightarrow$　$a=\frac{b}{2}$, $E=\frac{Ub}{\frac{b}{2}\cdot\frac{b}{2}}=\frac{4U}{b}$

3. 4pF　30V

$\frac{1}{\frac{1}{2C}+\frac{1}{C}}=\frac{2C^2}{2C+C}=\frac{2C}{3C}=\frac{2}{3}C$.



4. $\frac{(2\sqrt{2}-1)q^2}{32\pi\varepsilon_0 a^2}$　$\frac{(\sqrt{5}-\sqrt{2})q}{50\pi\varepsilon_0 a^2}$



$F=\frac{1}{4\pi\varepsilon_0}\frac{q^2}{(2a)^2}=\frac{q^2}{16\pi\varepsilon_0 a^2}$

$F=\frac{1}{4\pi\varepsilon_0}\frac{q^2}{(2\sqrt{2}a)^2}=\frac{q^2}{32\pi\varepsilon_0 a^2}$

$E_1=\frac{2q}{4\pi\varepsilon_0 a^2}=\frac{q}{2\pi\varepsilon_0 a^2}$　$E_2=\frac{2q}{4\pi\varepsilon_0 5a^2}\times\frac{1}{\sqrt{5}}=\frac{q}{10\sqrt{5}\pi\varepsilon_0 a^2}$

$E=\frac{\sigma}{\varepsilon_0}$, $\sigma=(E_1-E_2)\varepsilon_0=\frac{(5\sqrt{5}-1)q}{10\sqrt{5}\pi a^2}$

5. $\frac{2\pi\rho_e^2R^5}{27\varepsilon_0}$　$\frac{2\pi R^5\rho_e}{27\varepsilon}$　$\frac{2\pi\rho_e^2R^5}{45\varepsilon_0}$　$\frac{2\pi\rho_e^2R^5}{9\varepsilon}$

内: $w=\frac{1}{2}\varepsilon_0E^2$, $E=\frac{Q}{4\pi r^2\varepsilon_0}$

$W=\iiint\frac{1}{2}\varepsilon_0\frac{\rho^2}{16\pi^2\varepsilon_0^2r^4}dV$

$Q=\rho\cdot\frac{4}{3}\pi r^3$　$=\frac{\rho^2}{32\pi^2\varepsilon_0}\int_0^{2\pi}\int_0^{\pi}\int_0^R\frac{1}{r^2}dr\,d\varphi\,d\theta$

$D=\frac{Q}{4\pi r^2}=\frac{\rho\frac{4}{3}\pi r^3}{4\pi r^2}=\frac{\rho}{3}r$.　内: $w=\frac{1}{2}\varepsilon_0E^2=\frac{1}{2}\varepsilon_0\frac{\rho^2}{9\varepsilon_0^2}=\frac{\rho^2}{18\varepsilon}$

$E=\begin{cases}\frac{\rho}{3\varepsilon}r, & r<R\\ \frac{\rho}{3\varepsilon}, & r>R\end{cases}$　外: $w=\frac{1}{2}D\cdot E=\frac{\rho^2}{18\varepsilon}$

$D\times4\pi r^2=\int_0^{2\pi}\int_0^{\pi}\int_0^{R}\rho_e\,r^2\sin\theta\,d\theta\,d\varphi\,dr$

$=\frac{2\pi\times2}{3}R^3\rho_e=\frac{4}{3}\pi R^3\rho_e$

$D=\begin{cases}\frac{\rho_eR^3}{3r^2} & r>R\\ \frac{\rho_e r}{3} & r<R\end{cases}$

$E=\begin{cases}\frac{\rho_eR^3}{3r^2\varepsilon} & r>R\\ \frac{\rho_e r}{3\varepsilon_0} & r<R\end{cases}$

$W_{外}=\iiint\frac{1}{2}DE\,dV=\int_0^{2\pi}\int_0^{\pi}\int_R^{+\infty}\frac{\rho_eR^3}{3r^2}\times\frac{\rho_eR^3}{3r^2\varepsilon}r^2\sin\theta\,dr\,d\varphi\,d\theta$

$=\frac{\rho_e^2R^6}{9\varepsilon}4\pi\times\int_R^{+\infty}\frac{1}{r^2}dr=\frac{4\pi\rho_e^2R^5}{9\varepsilon}\times\frac{1}{2}$

$W_{内}=\iiint\frac{1}{2}\varepsilon_0E^2\,dV=\int_0^{2\pi}\int_0^{\pi}\int_0^{R}\frac{\rho_e^2r^2}{9\varepsilon_0}r^2\sin\theta\,dr\,d\varphi\,d\theta$

$=4\pi\times\frac{1}{5}r^5\Big|_0^R\frac{\rho_e^2}{9\varepsilon_0}=\frac{4\pi\rho_e^2R^5}{45\varepsilon}\times\frac{1}{2}$

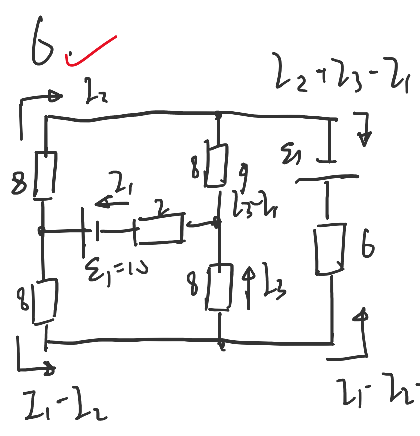
6. $\frac{1}{\sigma S}$　$\frac{I}{neS}$　$\frac{I^2l}{\sigma S}$

$J=\frac{I}{S}$, $j=\sigma E$.

$R=\rho\frac{l}{S}=\frac{l}{\sigma S}$　$I=\frac{dQ}{dt}=\frac{nesl}{dt}=nesv$.

$I^2R=\frac{I^2l}{\sigma S}$

7. 6　　1



$8I_2-8(I_3-I_1)+2I_1-10=0$

$2I_1-10+8(I_1-I_2)+8I_3=0$

$8I_3+8(I_3-I_1)+6(I_2+I_3-I_1)=10$

$16+6-14$

$\Rightarrow\begin{cases}5I_1+4I_2-4I_3=5\\5I_1-4I_2+4I_3=5\\-7I_1+3I_2+11I_3=5\end{cases}$　$I_1=1A$.　$I_2=I_3=\frac{3}{7}$　$-7+14I_2=5$.

二. 1. ×　　-3

2. √ $U\propto C$.

3. √ ×　$I=\frac{Q}{4\pi\varepsilon r}$

4. √

5. √

6. √

7. ×

8. ×

9. ×

10. √

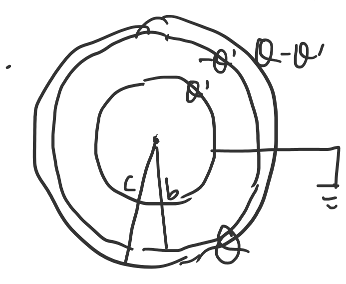
对E?

$E=\frac{\sigma}{\varepsilon}=\frac{Q}{\varepsilon S}$　$U=\frac{Q}{\varepsilon S}d$, $C=\frac{\varepsilon S}{d}\Rightarrow$ 67

1. ×　2. √ ×　3. × ×　4. √　5. ×　6. √　7. √　8. √　9. × √　10. × ×

$dU=\frac{dq}{4\pi\varepsilon r}$

三. 1.



(1) 设总为 Q'.　-5

$E=\begin{cases}\frac{1}{4\pi r^2\varepsilon_0}\frac{r^3}{a^3}Q', & r<a\\ \frac{1}{4\pi r^2\varepsilon_0}Q', & a<r<b\\ 0, & b<r<c\\ \frac{1}{4\pi r^2\varepsilon_0}Q, & r>c\end{cases}$

$\therefore U=\int_c^{\infty}\frac{Q}{4\pi r^2\varepsilon_0}dr+\int_b^c 0\,dr-\int_a^b\frac{Q'}{4\pi r^2\varepsilon_0}=0$

$\therefore\frac{Q}{4\pi\varepsilon_0}(-\frac{1}{r})\Big|_c^{\infty}+\frac{Q'}{4\pi\varepsilon_0}(\frac{1}{r})\Big|_a^b=0$

$Q\times\frac{1}{c}=-Q'(\frac{1}{b}-\frac{1}{a})$, $Q'=\frac{\frac{1}{c}}{\frac{1}{b}-\frac{1}{a}}Q$

(2) $\Delta U=\int_c^{\infty}\frac{Q}{4\pi r^2\varepsilon_0}=\int_c^{\infty}\frac{Q}{4\pi\varepsilon_0r^2}dr=\frac{Q}{4\pi\varepsilon_0c}$.

$C=\frac{Q}{\Delta U}=4\pi\varepsilon_0c$

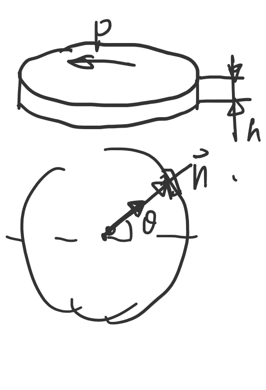
$U_{c\infty}=\int_c^{\infty}\frac{1}{4\pi r^2\varepsilon_0}Q\,dr=\frac{Q}{4\pi\varepsilon_0c}$　$C=\frac{4\pi\varepsilon_0a^3}{b^2-c^2}+4\pi\varepsilon_0c$

$C_外=\frac{Q_外}{U}=4\pi\varepsilon_0c$

$U_{ab}=\int_a^b\frac{r}{4\pi\varepsilon_0a^3}Q'dr=\frac{1}{2}r^2\Big|_a^b\frac{1}{4\pi\varepsilon_0a^3}\cdot\frac{\frac{1}{c}}{\frac{1}{b}-\frac{1}{a}}Q=\frac{b^2-a^2}{2}\cdot\frac{1}{4\pi\varepsilon_0a^3}\cdot\frac{ab}{(a-b)c}Q$

$C_{内}=\frac{Q_内}{U_内}=\frac{4\pi\varepsilon_0a^3}{\frac{1}{2}(b^2-a^2)}=\frac{8\pi\varepsilon_0a^3}{b^2-a^2}$

2.



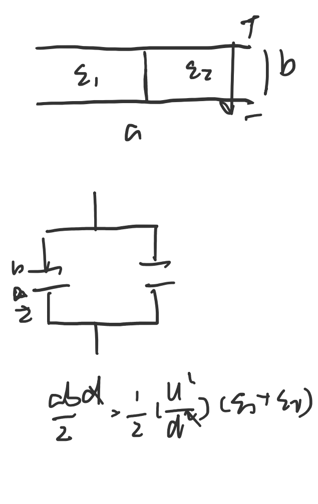
(1) $\sigma_{侧}=\vec{n}\cdot\vec{P}=P\cos\theta$

$\sigma_{顶}=\vec{n}\cdot\vec{P}=0$.

(2) $dE=\frac{\sigma dS}{4\pi\varepsilon_0R^2}$　$h<<R$, $\vec{E}\parallel$ 极

$E=2\int_0^{\pi}\frac{\sigma R\cos\theta\,d\theta}{4\pi\varepsilon_0R^2}h=\frac{P}{2\pi\varepsilon_0Rh}\int_0^{\pi}\cos\theta\,d\theta=\frac{Ph}{4\varepsilon_0R}$

3.



(1) $E=\frac{U}{d}$.　-15

$D_1=\varepsilon_1E=\frac{\varepsilon_1U}{d}$, $D_2=\frac{\varepsilon_2U}{d}$

(2) $D\times ab=Q=\sigma ab$,

$\sigma_{01}=\frac{\varepsilon_1U}{d}$, $\sigma_{02}=\frac{\varepsilon_2U}{d}$　1分

$P_1=\frac{U}{d}(\varepsilon_1-\varepsilon_0)$, $P_2=\frac{U}{d}(\varepsilon_2-\varepsilon_0)$

$\sigma_1'=\frac{U}{d}(\varepsilon_1-\varepsilon_0)$, $\sigma_2'=\frac{U}{d}(\varepsilon_2-\varepsilon_0)$

$\frac{abd}{2}\cdot\frac{1}{2}\frac{U^2}{d^2}(\varepsilon_1+\varepsilon_2)$

(3) $w=\frac{1}{2}\vec{D}\cdot\vec{E}$, $w_1=\frac{1}{2}\varepsilon_1(\frac{U}{d})^2$, $w_2=\frac{1}{2}\varepsilon_2(\frac{U}{d})^2$.

$W=\iiint w_1dV+\iiint w_2dV=\frac{abU^2}{4d}(\varepsilon_1+\varepsilon_2)$　$\frac{1}{2}\varepsilon\frac{U^2}{d^2}abd$

(4) 均匀极化, $\frac{\sigma_1'}{-\sigma_1'}$　$\frac{\sigma_2'}{-\sigma_2'}$　两部分有电荷作用.

没有任何 S.

(1) $E=\frac{U}{d}$.

$D_1=\varepsilon_1E=\frac{\varepsilon_1U}{d}$, $D_2=\varepsilon_2E=\frac{\varepsilon_2U}{d}$.

(2) 极上自由电荷面密度: $\sigma_{1+}=\vec{n}\cdot\vec{D}_1=\frac{\varepsilon_1U}{d}$　$\sigma_{1-}=-\frac{\varepsilon_1U}{d}$

$\sigma_{2+}=\frac{\varepsilon_2U}{d}$　$\sigma_{2-}=-\frac{\varepsilon_2U}{d}$

介质极化电荷面密度: $\vec{P}_1=D_1-\varepsilon_0E=(\varepsilon_1-\varepsilon_0)\frac{U}{d}$.

$\vec{P}_2=D_2-\varepsilon_0E=(\varepsilon_2-\varepsilon_0)\frac{U}{d}$.

$\sigma_{1+}'=\vec{n}_{01}\cdot(-\vec{P}_1)-0=-\vec{n}\cdot\vec{P}_1=(\varepsilon_1-\varepsilon_0)\frac{U}{d}$

$\sigma_{1-}'=-\vec{n}_{02}\cdot(-\vec{P}_1)-0=-\vec{n}\cdot\vec{P}_1=-(\varepsilon_1-\varepsilon_0)\frac{U}{d}$

(3) 并联, $C=C_1+C_2=\varepsilon_1C_0+\varepsilon_2C_0=(\varepsilon_1+\varepsilon_2)\frac{\varepsilon_0ab}{d}$

$E=\frac{\sigma}{\varepsilon_0}=\frac{\sigma_0}{\varepsilon_0}$, $E=\frac{Q}{\varepsilon_0S}$, $U=\frac{Qd}{\varepsilon_0S}$. $C=\frac{\varepsilon_0S}{d}$.

$W=\frac{1}{2}CU^2=\frac{(\varepsilon_1+\varepsilon_2)\varepsilon_0ab}{2d}U^2$.

(4) $W=\frac{1}{2}CU^2=\frac{1}{2}(C_1+C_2)U^2=\frac{1}{2}(\frac{\varepsilon_1\varepsilon_0\frac{a}{2}b}{d}+\frac{\varepsilon_2\varepsilon_0\frac{a}{2}b}{d})U^2$

$\frac{\partial W}{\partial x_1}=\frac{1}{2}\frac{U^2}{d}\varepsilon_1\varepsilon_0b$, $\frac{\partial W}{\partial x_2}=\frac{1}{2}\frac{U^2}{d}\varepsilon_2\varepsilon_0b$.

虚功原理. $C=\frac{\varepsilon_1(\frac{a}{2}+x)b}{d}+\frac{\varepsilon_2(\frac{a}{2}-x)b}{d}=C_0+\frac{(\varepsilon_1-\varepsilon_2)xb}{d}$

$F=\frac{2(\varepsilon_1-\varepsilon_2)bU^2}{d}$