知识点

2022年5月26日 星期四　　上午9:58

交流电路

一、基本概念和描述方法.

1.基本概念

电路中的电源电动势、电流、电压随时间周期性变化.

电路路途是非稳态的. 但可采用似稳近似（$l \ll \lambda$ 判据式）.

2.描述方法

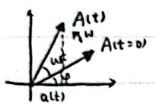
①函数描述：u. 或 i. e 均可写成正余弦形式.

$u(t) = V_m \cos(\omega t + \varphi)$. 频率和周期. $\omega = \frac{2\pi}{T}$.

峰值 V_m 和有效值 $\frac{1}{\sqrt{2}}V_m$（交流电表读数）.

相位和相位差. $\omega t + \varphi$ 为相位, φ 为初相, $\varphi_1 - \varphi_2$ 相位差.

②矢量描述：$a(t) = A\cos(\omega t + \varphi)$



$a(t) = \vec{A} \cdot \vec{e_n} = A\cos(\omega t + \varphi)$

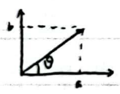
$a_1(t) = A_1\cos(\omega_1 t + \varphi_1)$. $a_2(t) = A_2\cos(\omega t + \varphi_2)$, 合成 $A\cos(\omega t + \varphi) = a_1(t) + a_2(t)$.

则 $\begin{cases} A\cos\varphi = A_1\cos\varphi_1 + A_2\cos\varphi_2 \\ A\sin\varphi = A_1\sin\varphi_1 + A_2\sin\varphi_2 \end{cases}$, $\Rightarrow A = [A_1^2 + A_2^2 + 2A_1A_2\cos(\varphi_2 - \varphi_1)]^{\frac{1}{2}}$

$\tan\varphi = \frac{A_1\sin\varphi_1 + A_2\sin\varphi_2}{A_1\cos\varphi_1 + A_2\cos\varphi_2}$. $\vec{A} = \vec{A_1} + \vec{A_2}$

矢量相等 $\Leftrightarrow$ x 分量相等.

③复数描述：$\tilde{A} = a + jb$, $a = Re(\tilde{A})$, $b = Im(\tilde{A})$



$\vec{A} = a\vec{e_x} + b\vec{e_y}$, $|\tilde{A}| = \sqrt{a^2+b^2}$, $\theta = \arctan\frac{b}{a}$, $\tilde{A} = A(\cos\theta + j\sin\theta) = Ae^{j\theta}$

$\theta = \omega t + \varphi$, $\tilde{A} = Ae^{j(\omega t + \varphi)}$, $a(t) = Re(\tilde{A}) = A\cos(\omega t + \varphi)$

$Re(\tilde{A}) = Re(\tilde{A_1} + \tilde{A_2})$, $\tilde{A} = \tilde{A_1} + \tilde{A_2}$, 旋转矢量复数相等 $\Leftrightarrow$ 实部相等

$\tilde{V} = V_m e^{j(\omega t + \varphi)}$, $u(t) = Re(\tilde{V}) = V_m\cos(\omega t + \varphi)$. $V_m = \sqrt{2}V$（有效值）,

则 $\tilde{V} = \sqrt{2}Ve^{j(\omega t + \varphi)} = \sqrt{2}\dot{V}e^{j\omega t}$, $\dot{V} = Ve^{j\varphi}$, 复有效值.

二、交流电路的复数解法.

1.交流电路的基本方程

单回路：$e = iR + \frac{1}{C}\int i\,dt + L\frac{di}{dt} + M\frac{di'}{dt}$. 多回路：$\sum I_{入} = \sum i_{出}$. $\sum e = \sum iR + \sum\frac{1}{C}\int i\,dt + \sum L\frac{di}{dt} + \sum M\frac{di'}{dt}$.

当 R. C. L. M 为常量时, 为线性电路, 若干个信号源的 e_i. i_i 可以叠加.

$e_1 + e_2 = (i_1 + i_2)R + \frac{1}{C}\int(i_1 + i_2)dt + L\frac{d(i_1 + i_2)}{dt} + M\frac{d(i_1' + i_2')}{dt}$. 傅立叶分析化为简单信号.

2.电路方程的复数形式.

$Re(\tilde{\varepsilon}) = R \cdot Re(\tilde{I}) + \frac{1}{C}\int Re(\tilde{I})dt + L\frac{d}{dt}Re(\tilde{I}) + M\frac{d}{dt}Re(\tilde{I}') = Re(R\tilde{I} + \frac{1}{C}\int\tilde{I}dt + L\frac{d\tilde{I}}{dt} + M\frac{d\tilde{I}'}{dt})$

复数微分、积分 → 对实部和虚部分别微积分.

又 $\tilde{I} = I_m e^{j(\omega t + \varphi)}$, $\int\tilde{I}dt = \frac{1}{j\omega}\tilde{I}$, $\frac{d\tilde{I}}{dt} = j\omega\tilde{I}$. $\therefore Re(\tilde{\varepsilon}) = Re(R\tilde{I} + \frac{1}{j\omega C}\tilde{I} + j\omega L\tilde{I} + j\omega M\tilde{I}')$.

$\therefore \tilde{\varepsilon} = R\tilde{I} + \frac{1}{j\omega C}\tilde{I} + j\omega L\tilde{I} + j\omega M\tilde{I}'$. $\dot{\varepsilon} = R\dot{I} + \frac{1}{j\omega C}\dot{I} + j\omega L\dot{I} + j\omega M\dot{I}$. $\sum\dot{I}_{入} = \sum\dot{I}_{出}$.

3. 交流电路元件的复阻抗. → 分析方式与稳恒电路的电阻相同.

电阻、电容、电感都有 $\dot{V} = \dot{I}\tilde{Z}$, $\tilde{Z} = \frac{\dot{V}}{\dot{I}} = Ze^{j\varphi}$. $\tilde{Z} = \tilde{Z}_R + \tilde{Z}_L + \tilde{Z}_M + \tilde{Z}_C = R + j\omega L + j\omega M + \frac{1}{j\omega C}$, $j = e^{j\frac{\pi}{2}}$.

三、交流电的功率.

1. 瞬时功率 $p(t) = u(t)i(t) = V_m I_m\cos\omega t\cos(\omega t + \varphi) = \frac{1}{2}V_m I_m\cos\varphi + \frac{1}{2}V_m I_m\cos(2\omega t + \varphi)$.

2. 平均功率 $P = \bar{p} = \frac{1}{T}\int_0^T p(t)dt = \frac{1}{2}V_m I_m\cos\varphi = VI\cos\varphi$. 纯电容. 纯电感不耗能.

3. 视在功率 额定电压与额定电流的乘积 $S = VI$.

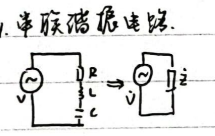
平均功率 P 称为有功功率、实在功率, $P = S\cos\varphi$, $\cos\varphi$ 为功率因素.

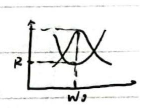
4.平均功率计算.

$\dot{V} = Ve^{j\varphi_u}$. $\dot{I} = Ie^{j\varphi_i}$. $\dot{V}\bar{I} = VIe^{j\varphi}$. $\bar{V}\dot{I} = VIe^{-j\varphi}$. $P = Re(\dot{V}\bar{I}) = Re(\bar{V}\dot{I}) = \frac{1}{2}(\dot{V}\bar{I} + \bar{V}\dot{I})$

四. 交流电路分析举例.

1. 串联谐振电路.





$\tilde{Z} = \tilde{Z}_R + \tilde{Z}_C + \tilde{Z}_L = R + j\omega L + \frac{1}{j\omega C} = R + j\omega L(1 - \frac{\omega_0^2}{\omega^2})$, $\omega_0 = \frac{1}{\sqrt{LC}}$.

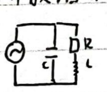
$\dot{V} = \dot{I}\tilde{Z}$, $I = \frac{V}{Z}$, $\varphi_I = -\varphi_Z$.（电压初相为零）. 谐振 $\omega = \omega_0$

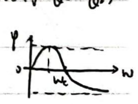
$I_{max} = \frac{V}{R}$, $Z_{min} = R$. 品质 $Q = \frac{1}{R\omega_0 C} = \frac{\omega_0 L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$. $Q = \frac{Z_C}{R} = \frac{Z_L}{R} = \frac{V_C}{V_R} = \frac{V_L}{V_R}$.

频率 f 变化 Δf, 若使 $Z = \sqrt{2}Z_{min}$ 或 $I = \frac{1}{\sqrt{2}}I_{max}$, $2\Delta f$ 为通频带宽度.

$(\frac{\omega_0}{\omega})^2 = (1 + \frac{\Delta\omega}{\omega_0})^{-2} \approx 1 - \frac{2\Delta\omega}{\omega_0}$, $\frac{\Delta\omega}{\omega_0} = \frac{R}{2\omega_0 L} = \frac{1}{2Q}$, $2\Delta f = \frac{\Delta\omega}{\pi} = \frac{f_0}{Q}$. $Q = \frac{2\pi L}{T_0 R} = 4\pi\frac{\frac{1}{2}LI^2}{I^2RT_0}$, 储能.

2. 并联谐振电路.



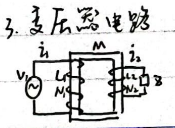


$\tilde{Z} = (\frac{1}{\tilde{Z}_C} + \frac{1}{\tilde{Z}_R + \tilde{Z}_L})^{-1} = \frac{R + j\omega L(1 - \frac{\omega^2}{\omega_0^2} - \frac{CR^2}{L})}{(1 - \frac{\omega^2}{\omega_0^2})^2 + \omega^2C^2R^2}$. $\omega_c = \omega_0\sqrt{1 - \frac{1}{Q^2}}$, $\omega_c' = \omega_0(\sqrt{1 + \frac{2}{Q^2}} - \frac{1}{Q^2})^{\frac{1}{2}}$.

谐振角频率 $\omega_c = \omega_0$, $Z_{max} \approx Q^2R$, $I_{min} = \frac{V}{Q^2R}$.

$\varphi_Z = \arctan[\frac{\omega L}{R}(1 - \frac{\omega^2}{\omega_0^2} - \frac{CR^2}{L})]$. $\omega = \frac{\omega_0}{\sqrt{3}}$, $\varphi_{Zm} = \arctan[\frac{2Q}{3\sqrt{3}}(1 - \frac{1}{Q^2})^{\frac{3}{2}}]$.

3. 变压器电路



$\dot{I}_1\tilde{Z}_1 - \dot{I}_2\tilde{Z}_M = \dot{V}_1$, $-\dot{I}_1\tilde{Z}_M + \dot{I}_2\tilde{Z}_2 = 0$. $\tilde{Z}_1 = R_1 + j\omega L_1$, $\tilde{Z}_2 = R_2 + j\omega L_2 + \tilde{Z}$. $\tilde{Z}_M = j\omega M$.

$\dot{I}_1 = \frac{\dot{V}_1\tilde{Z}_2}{\tilde{Z}_1\tilde{Z}_2 - \tilde{Z}_M^2}$, $\dot{I}_2 = \frac{\dot{V}_1\tilde{Z}_M}{\tilde{Z}_1\tilde{Z}_2 - \tilde{Z}_M^2}$. $\frac{\dot{I}_1}{\dot{I}_2} = \frac{\tilde{Z}_M}{\tilde{Z}_2}$, $\frac{\dot{V}_2}{\dot{V}_1} = \frac{\dot{I}_2\tilde{Z}}{\dot{V}_1} = \frac{\tilde{Z}\tilde{Z}_M}{\tilde{Z}_1\tilde{Z}_2 - \tilde{Z}_M^2}$.

理想变压器（$M^2 = L_1L_2$, $R_1 = R_2 = 0$）. $\frac{\dot{I}_2}{\dot{I}_1} = \frac{I_2}{I_1} = \frac{N_1}{N_2}$, $\frac{V_2}{V_1} = \frac{N_2}{N_1}$. $\tilde{Z}_1' = \frac{L_1}{L_2}\tilde{Z} = \frac{N_1^2}{N_2^2}\tilde{Z}$. $P_1 = P_2$.