

交流电

2022年5月26日 星期四 上午10:51

一. 描述法与复数表达式.

1. 描述法.

① 函数

② 矢量.

- 正弦交流量, 幅值为峰值, 夹角为初相位 φ .

任意 t , 夹角 $\omega t + \varphi$, 瞬时值为投影 $A \cos(\omega t + \varphi)$.

基准矢量, 参照其他矢量.

③ 复数.

2. 阻抗与幅角.

一个元件 $\begin{cases} u = U_m \cos(\omega t + \varphi_u) \\ i = I_m \cos(\omega t + \varphi_i) \end{cases}$

则定义 阻抗 $Z = \frac{U_m}{I_m} \Rightarrow$ 描述该元件.

幅角 $\varphi = \varphi_u - \varphi_i$.

① 电阻: $U_m \cos(\omega t + \varphi_u) = I_m R \cos(\omega t + \varphi_i)$

$$\frac{U_m}{I_m} = R, \varphi_u - \varphi_i = 0$$

② 电容: $q(t) = Q_m \cos(\omega t + \varphi_q)$

$$\Rightarrow \begin{cases} I(t) = \omega Q_m \cos(\omega t + \varphi_q - \frac{\pi}{2}) = I_m \cos(\omega t + \varphi_i) \\ U(t) = \frac{Q_m}{C} \cos(\omega t + \varphi_q) = U_m \cos(\omega t + \varphi_u) \end{cases}$$

$$\therefore Z = \frac{U}{I} = \frac{1}{\omega C}$$

$$\varphi = \varphi_u - \varphi_i = \varphi_q - (\varphi_q - \frac{\pi}{2}) = \frac{\pi}{2}$$

③ 电感: $i(t) = I_m \cos(\omega t + \varphi_i)$

$$u(t) = L \frac{di(t)}{dt} = I_m \omega L \cos(\omega t + \varphi_i + \frac{\pi}{2})$$

$$Z = \frac{U_m}{I_m} = \omega L$$

$$\varphi = \varphi_u - \varphi_i = \frac{\pi}{2}$$

3. 交流电功率.

$$i = I_m \cos(\omega t + \varphi_i)$$

$$u = I_m Z \cos(\omega t + \varphi_i + \varphi)$$

$$\text{瞬时功率 } p = iu = I^2 Z \cos(\omega t + \varphi_i) \cos(\omega t + \varphi_i + \varphi)$$

$$\text{平均功率 } P = \langle p \rangle = I^2 Z \cos \varphi$$

视在功率 $S = IU$ 单位.

$$\text{功率因数 } \cos \varphi = \frac{P}{S}$$

$$\text{有功电阻 } r = Z \cos \varphi$$

$$P = I^2 r = S \cos \varphi$$

二. 矢量法求 Z

0. 洛伦兹-亥维格夫定律.

矢量法. $\vec{U}_m = A \cos(\omega t + \varphi)$

1. 串联情况

① RL 串联.

$$U(t) = U_R(t) + U_L(t)$$

$$\begin{cases} U = IZ = \sqrt{U_R^2 + U_L^2} \\ \tan \varphi = \frac{U_L}{U_R} \end{cases} \Rightarrow \begin{cases} Z = I \sqrt{R^2 + (\omega L)^2} \\ \varphi = \arctan \frac{Z_L}{Z_R} = \arctan \frac{\omega L}{R} \end{cases}$$

② RC 串联

$$\begin{cases} U = \sqrt{U_R^2 + U_C^2} \\ \tan \varphi = -\frac{U_C}{U_R} \end{cases} \Rightarrow \begin{cases} Z = \\ \varphi = \end{cases}$$

③ RLC 串联.

$$Z = \sqrt{R^2 + (Z_L - Z_C)^2}$$

$$\varphi = \arctan \frac{Z_L - Z_C}{Z_R}$$

2. 共振.

$$\textcircled{1} Z = \sqrt{R^2 + (Z_L - Z_C)^2}$$

$Z_L = Z_C$ 时 阻抗最小, 电流最大.

② 电流与电压同相.

$$\text{幅角 } \varphi = \frac{Z_L - Z_C}{R}$$

③ 储能. 耗能.

功率.

④ 电压分配

L 与 C 两端电压相同.

$$\frac{U_L}{U_R} = \frac{I Z_L}{I R} = \frac{\omega L}{R} = \frac{1}{R} \sqrt{\frac{L}{C}} = \frac{U_C}{U_R}$$

3. 并联情况

① RL

$$Z = \frac{1}{\sqrt{\frac{1}{Z_R^2} + \frac{1}{Z_C^2}}} = \frac{1}{\sqrt{\frac{1}{R^2} + \frac{1}{\omega L^2}}}$$

② RC

$$Z = \frac{1}{\sqrt{\frac{1}{Z_R^2} + \frac{1}{Z_C^2}}}$$

③ RLC

$$Z = \frac{1}{\sqrt{\frac{1}{Z_R^2} + \frac{1}{Z_C^2} - \frac{1}{Z_L^2}}}$$

例.

三. 复数法求 Z .

1. 复数表示.

$$\textcircled{1} u(t) = U_m \cos(\omega t + \varphi_u)$$

$$\Rightarrow \hat{u}(t) = U_m [\cos(\omega t + \varphi_u) + i \sin(\omega t + \varphi_u)] = U_m e^{i(\omega t + \varphi_u)}$$

$$\textcircled{2} \text{ 复有效值 } \hat{U} = U e^{i\varphi_u}$$

$$\text{复峰值 } \tilde{U}_m = U_m e^{i\varphi_u}$$

$$\text{复阻抗 } \tilde{Z} = \frac{\tilde{U}}{I} = Z e^{i\varphi}$$

$$\tilde{Z}_R = R, \tilde{Z}_L = j\omega L, \tilde{Z}_C = \frac{1}{j\omega C}$$

有功电阻 $r = Z \cos \varphi$ (实部)

电抗 $x = Z \sin \varphi$ (虚部)

③ 与实值一一对应, 亥维格夫.

2. 电路中

① 串联: 与实值同.

② 功率.