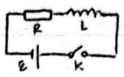
知识点

2022年5月24日 星期二　　下午2:53

磁能.

一. 载流线圈的磁能.

1. 一个载流线圈中的磁能



$\varepsilon - L\frac{dI}{dt} = IR$，$\varepsilon I dt = LIdI + I^2Rdt = -\varepsilon_i I dt + I^2Rdt$.

电源做功→焦耳热+反抗感应电动势做功（磁场能）

磁场能 $W_m = \int_0^I LIdI = \frac{1}{2}LI^2 = \frac{1}{2}I\Phi_m$

2. N个载流线圈系统的磁能.

第i个线圈. $\varepsilon_i = -L_i\frac{dI_i}{dt} - \sum_{k=1,k\neq i}^{N} M_{ik}\frac{dI_k}{dt}$，$dA_i' = L_iI_idI_i + \sum_{k=1,k\neq i}^{N} M_{ki}I_idI_k$

$dA' = \sum_{i=1}^N dA_i = \sum_{i=1}^N L_iI_idI_i + \sum_{i,k=1,k\neq i}^{N} M_{ki}I_idI_k = \frac{1}{2}\sum_{i,k=1,i\neq k}^{N} M_{ik}d(I_iI_k) + \sum_{i=1}^N I_i^2dI_i$

$W = A' = \frac{1}{2}\sum_{i,k=1,i\neq k}^{N} M_{ik}I_iI_k + \frac{1}{2}\sum_{i=1}^N L_iI_i^2 = \frac{1}{2}\sum_{i,k=1}^{N} M_{ik}I_iI_k$. 互感磁能+自感磁能

$= \frac{1}{2}\sum_{i=1}^N I_i\Phi_i$，$\Phi_i = \sum_{k=1}^N \Phi_{ki} = \sum_{k=1}^N M_{ik}I_k$（第k个线圈的B通过第i个线圈的磁通）

二. 载流线圈在外磁场中的磁能.

对两个载流线圈组成的系统，$W_m = \frac{1}{2}L_1I_1^2 + \frac{1}{2}L_2I_2^2 + M_{12}I_1I_2$（自感磁能+互感磁能）

互能 $W_{12} = M_{12}I_1I_2 = \Phi_{12}I_2 = I_2\iint_{S_2}\vec{B}_1(\vec{r}_2)\cdot d\vec{S}$ 线圈1的磁场通过线圈2产生的磁通.

均匀外磁场、小载流线圈，$\vec{B}$为定，$W_{12} = \vec{B}\cdot(I_2\vec{S}) = \vec{m}\cdot\vec{B}$.

N个线圈. 均匀外场 $W_m = \vec{B}\cdot(\sum_{k=1}^N I_k\vec{S}_k) = \vec{m}_t\cdot\vec{B}$，不均匀外场 $W_m = \sum_{k=1}^N I_k\iint_{S_k}\vec{B}(\vec{r})\cdot d\vec{S}$.

三. 磁场的能量和磁能密度.

螺绕环磁导率μ，周长为l，截面积S，匝数N，电流I，则$B = \mu nI = \mu\frac{N}{l}I$.

自感系数 $L = \frac{NSB}{I} = \frac{NS\mu nI}{I} = \mu n^2V$，$V = Sl$为螺绕环体积.

$\therefore W_m = \frac{1}{2}LI^2 = \frac{1}{2}V\mu n^2I^2 = \frac{1}{2}VBH$. 磁能密度 $w_m = \frac{W_m}{V} = \frac{1}{2}\vec{B}\cdot\vec{H}$. $W_m = \iiint_V w_m dV$.

$H_1 = \frac{1}{2\pi r}\frac{I}{\pi r_1^2}\pi r^2 = \frac{Ir}{2\pi r_1^2}$，$B_1 = \mu_0H_1 = \frac{\mu_0Ir}{2\pi r_1^2}$，$w_{m1} = \frac{\mu_0I^2r^2}{8\pi^2r_1^4}$，$W_{m1} = \int_0^{r_1}\int_0^{2\pi}\int_0^l w_{m1}rdrd\varphi dl = \frac{\mu_0lI^2}{16\pi}$

$H_2 = \frac{I}{2\pi r}$，$B_2 = \frac{\mu I}{2\pi r}$，$w_{m2} = \frac{\mu I^2}{8\pi^2r^2}$，$W_{m2} = \int_{r_1}^{r_2}\int_0^{2\pi}\int_0^l w_{m2}rdrd\varphi dz = \frac{\mu lI^2}{4\pi}\ln(\frac{r_2}{r_1})$

$W_m = W_1 + W_2$，$L = \frac{2W_m}{I^2}$，$L_0 = \frac{L}{l}$

四. 非线性磁介质及磁损耗.

N匝螺绕环填充M的磁介质. $d\psi = NSdB$，$dA' = -\varepsilon Idt = I\frac{d\psi}{dt}dt = NSIdB$.

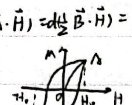
磁场强度 $\oint_l\vec{H}\cdot d\vec{l} = Hl = NI$，$\therefore dA' = NS\frac{Hl}{N}dB = V\cdot HdB$，单位体积 $dA' = \vec{H}\cdot d\vec{B}$.

对各向异性介质，$\vec{B} = \mu_0(\vec{H} + \vec{M})$，$dA' = d(\frac{\mu_0H^2}{2}) + \mu_0\vec{H}\cdot d\vec{M}$. $M_i = \sum_{j=1}^3 \chi_{ij}H_j$，各向同性 $\mu_0\vec{H}\cdot d\vec{M} = d(\frac{\mu_0}{2}\vec{M}\cdot\vec{H})$

$\mu_0\vec{M}\cdot\vec{H}$为磁化能密度，磁化功全部转换为介质的磁化能.

$dA' = d(\frac{\mu_0}{2}H^2 + \frac{\mu_0}{2}\vec{M}\cdot\vec{H}) = d(\frac{1}{2}\vec{B}\cdot\vec{H}) = dw_m$，电源做功 = 磁能 = 宏观磁能密度$\frac{\mu_0H^2}{2}$ + 磁化磁能$\frac{\mu_0\vec{M}\cdot\vec{H}}{2}$

对非线性介质



$A' = \oint dA' = \oint\mu_0HdM$. 磁滞损耗→热能.

五. 利用磁能求磁力.

1. N个线圈组成电流系统，其中一个线圈作一虚位移$\delta\vec{r}$，其他电流不变.

$\delta A = \vec{F}\cdot\delta\vec{r} = F_x\delta x + F_y\delta y + F_z\delta z$ 磁力$\vec{F}$做功. 设线圈电流不变的维持需电源做功$\delta A'$.

则系统磁能变化 $\delta W_m = \delta A' - \delta A$.

其中第i个线圈 $\delta A' = -\varepsilon_iI_idt = I_i\frac{d\Phi_i}{dt}dt = I_id\Phi_i$ 因受力线圈移动而Φ_i变化，则 $\delta A' = \sum_{i=1}^N I_id\Phi_i$.

又系统磁能 $(\delta W_m)_I = \frac{1}{2}\sum_{i=1}^N I_id\Phi_i$，$\therefore\delta A' = 2(\delta W_m)_I$，$\delta A = (\delta W_m)_I$. 磁力做功 = 磁能增加.

$\therefore\vec{F} = (\nabla W_m)$，$F_x = \frac{\partial(W_m)_I}{\partial x}$. 将I作为常数

若设维持各线圈磁通不变，则无感应电动势，电源不做功，$-\delta A = (\delta W_m)_\Phi$. 磁力做功，$W_m$减少.

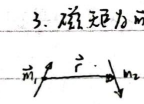
$\therefore\vec{F} = -(\nabla W_m)$，$F_x = \frac{-\partial(W_m)_\Phi}{\partial x}$. 将$\Phi$作为常数.

2. 将位移δr变化为角位移$\delta\theta$，力矩 $L_\theta = (\frac{\partial W_m}{\partial\theta})_I$ 或 $-(\frac{\partial W_m}{\partial\theta})_\Phi$.

$W_m = \vec{m}\cdot\vec{B} = mB\cos\theta$，若I不变，则$\vec{m}$不变，$\vec{F} = [\nabla(\vec{m}\cdot\vec{B})]_m = (\vec{m}\cdot\nabla)\vec{B} + \vec{m}\times(\nabla\times\vec{B}) = (\vec{m}\cdot\nabla)\vec{B}$.

$\therefore\vec{L}_\theta = (\frac{\partial W_m}{\partial\theta})_m\vec{e}_\theta = -mB\sin\theta\,\vec{e}_\theta = \vec{m}\times\vec{B}$.

3. 磁矩为$\vec{m}$的粒子在外磁场$\vec{B}$中的势能 $W_m = -\vec{m}\cdot\vec{B}$，其受磁力 $\vec{F} = -(\frac{\partial W_m}{\partial\theta})_m$，力矩 $\vec{L} = -(\frac{\partial W_m}{\partial\theta})_m$.



$\vec{B}_1 = \frac{-\mu_0\vec{m}_1}{4\pi r^3} + \frac{3\mu_0}{4\pi r^3}(\vec{m}_1\cdot\vec{e}_r)\vec{e}_r$　$W_m = \vec{m}_2\cdot\vec{B}_1 = \vec{m}_1\cdot\vec{B}_2 = -\frac{\mu_0}{4\pi r^3}(\vec{m}_1\cdot\vec{m}_2) + \frac{3\mu_0}{4\pi r^3}(\vec{m}_1\cdot\vec{e}_r)(\vec{m}_2\cdot\vec{e}_r)$.

$\vec{F}_{12} = (\nabla W_m)_m = \frac{3\mu_0\vec{e}_r}{4\pi r^4}(\vec{m}_1\cdot\vec{m}_2 - 5m_{1r}m_{2r}) + \frac{3\mu_0}{4\pi r^4}(m_{2r}\vec{m}_1 + m_{1r}\vec{m}_2)$.

电路中变化时：$I^2R + \frac{d}{dt}(\frac{Q^2}{2C}) = P$

$I^2R + \frac{d}{dt}(\frac{1}{2}LI^2) = P$.

磁场建立过程中外界抵抗 ε感 做的功.

单个载流线圈：$A = -\int I\varepsilon dt = \int Id\Phi$

$= \int_0^{I_0} ILdI = \frac{1}{2}LI_0^2 = \frac{1}{2}I\Phi$

载流线圈系统：$\Phi_i = \sum_{j=1}^N \Phi_{ij} = \sum_{j=1}^N M_{ij}I_j$.

$\varepsilon_i = -\frac{d\Phi_i}{dt} = -\sum_{j=1}^N M_{ij}\frac{dI_j}{dt}$.

自能与互能：$\frac{1}{2}L_iI_i^2 + \frac{1}{2}L_jI_j^2 + MI_iI_j$.

线圈在外场中的磁能：$W = I\Phi = I\iint\vec{B}\cdot d\vec{S}$

$= I\vec{B}\iint d\vec{S}$

$= I\vec{S}\cdot\vec{B} = mB$.

小载流线圈外场中势能：$U = -\vec{m}\cdot\vec{B}$.

任意电流分布的磁能：$W = \frac{1}{2}\sum I_i\Phi_i$

$= \frac{1}{2}\sum_j \Delta Q_1\oint\vec{A}\cdot d\vec{l}$

$= \frac{1}{2}\iiint\vec{j}\cdot\vec{A}dV$

螺线管的磁能：$W = \frac{1}{2}I\Phi = \frac{1}{2}INBS$

$= \frac{1}{2}Inl\mu_0nIS = \frac{1}{2}\mu_0n^2I^2V$

$= \frac{B^2}{2\mu_0}V$.

能量密度 $w = \frac{B^2}{2\mu_0}$.