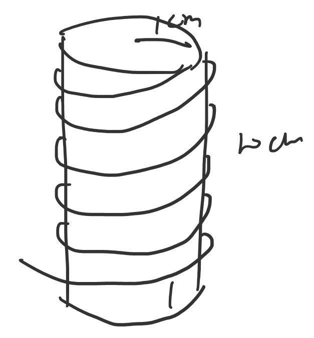
课后题

2022年6月2日 星期四 上午9:02

1.



$B=\mu_0 n I=\mu_0\frac{N}{l}I$

$L=\frac{NBS}{I}=\frac{N^2\mu_0 S}{l}=\frac{1000^2\times\mu_0\times\pi\times\left(\frac{0.01}{2}\right)^2}{0.1}$

$R=1000\times\pi\,0.01\times 247\,\Omega$

(1) $IR+L\frac{dI}{dt}=\varepsilon.$ 设 $\frac{R}{L}dt=-\frac{1}{I}dI.$

得 $I=\frac{\varepsilon}{R}.$ $\quad -\frac{R}{L}t=\ln I.\quad I=Ce^{-\frac{R}{L}t}$

$I(t)=\frac{\varepsilon}{R}(1-e^{-\frac{R}{L}t})$ → 初始 $I=0.$

$\frac{dI}{dt}=\frac{\varepsilon}{R}e^{\frac{R}{L}t}\times\frac{R}{L}=\frac{\varepsilon}{L}e^{\frac{R}{L}t}.\quad t=0,\ \frac{\varepsilon}{L}.$

(2) $t\to+\infty,\quad I=\frac{\varepsilon}{R}.$

(3) $\tau=\frac{L}{R}=$

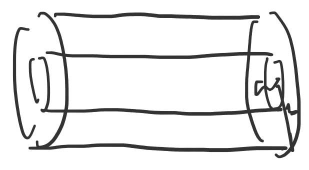
$\frac{\varepsilon}{R}-\frac{\varepsilon}{R}e^{-\frac{R}{L}t}=\frac{1}{2}\frac{\varepsilon}{R},\quad \frac{1}{2}=e^{-\frac{R}{L}t}$

$\ln\frac{1}{2}=-\frac{R}{L}t,\quad t=\frac{L}{R}\ln 2=$

(4) $W=\frac{1}{2}LI^2=\frac{1}{2}L\cdot\frac{\varepsilon^2}{R^2}=$

$W_m=\frac{W}{V}=$

2. $2b$



$B=\begin{cases}0 & 0<r<a\\ \frac{\mu_0}{2\pi r}I & a<r<b\\ 0 & r>b\end{cases}$

$w=\frac{1}{2\mu_0}\frac{\mu_0^2}{4\pi^2r^2}I^2,\ a<r<b.$

$W=\int_0^{2\pi}d\theta\int_a^b r w\,dr=\int\frac{\mu_0}{8\pi^2 r}I^2dr=\frac{\mu_0 I^2}{8\pi^2}\ln\frac{b}{a}.$

$W=\frac{1}{2}LI^2,\ L=\frac{2W}{I^2}=\frac{\mu_0}{4\pi}\ln\frac{b}{a}$

(2) $B=\frac{\mu_0}{2\pi r}I.\ a<r<2b.\quad W=\frac{\mu_0}{4\pi}\ln\frac{2b}{a}$

$\Delta W=\frac{\mu_0}{4\pi}(\ln\frac{2b}{a}-\ln\frac{b}{a})=\frac{\mu_0}{4\pi}\ln 2$

(3) $W=\frac{\mu_0}{4\pi}\ln 2,\quad W_外=\frac{\mu_0}{2\pi}\ln 2.$

$W_外+W=\Delta W$

$W_外=W+\Delta W.$

$i=\frac{I}{2\pi r}.$

受力 $iBds=\frac{i^2\mu_0}{2}$

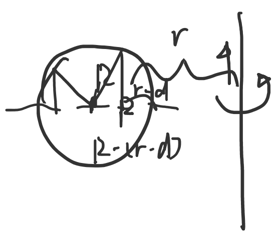
$A=\int_b^{2b}dr\iint iBds$

$=\frac{\mu_0 I^2}{4\pi}\ln 2$

这和方向与压力方向一致.

3.

$W_{12}=M I_1 I_2$



$B=\frac{\mu_0}{2\pi r}I_2$

$R^2-(r^2-2r(d+R)+(R+d)^2)$

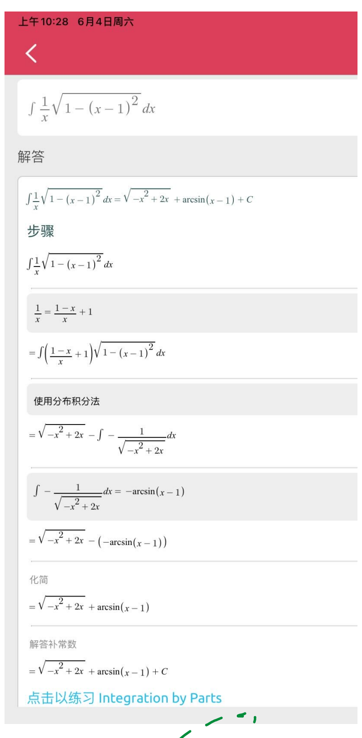
$\Phi=\int\vec{B}\cdot d\vec{S}$

$=\int_d^{d+R}\frac{\mu_0}{2\pi r}I_2\sqrt{R^2-(R-r+d)^2}\,dr$

$+\int_{d+R}^{d+2R}\frac{\mu_0}{2\pi r}I_2\sqrt{R^2-(r-d-R)^2}\,dr$

$=\int_d^{d+2R}\frac{\mu_0}{2\pi r}I_2\sqrt{2Rd+d^2-r^2-2(d+R)r}\,dr$

$=\frac{\mu_0 I_2}{2\pi}\int_d^{d+2r}\frac{1}{r}\sqrt{R^2-(r-(d+R))^2}\,dr$

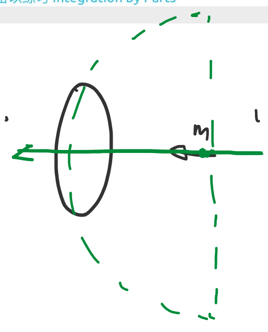


$=\mu_0 I_2(d-\sqrt{d^2-R^2})$

$\therefore W=MI_1I_2=\frac{\Phi}{I_2}I_1I_2=\Phi I_1$

$=\mu_0 I_1 I_2(d-\sqrt{d^2-R^2}).$

4.



(1) $W_0=\frac{1}{2}LI^2.$

对磁偶极子 m 为原点.

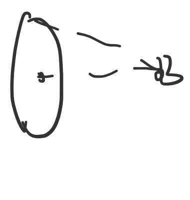
$r=\sqrt{b^2+z^2}\quad B_{mr}=\frac{\mu_0 m\cos\theta}{2\pi r_m^3}.$

$\Phi_m=\iint B_{mr}r^2\sin\theta\,d\theta\,d\varphi.$

$=\frac{\mu_0 m}{2r_m}\sin^2\theta_m=\frac{\mu_0 m b^2}{2(b^2+z^2)^{\frac{3}{2}}}.$

$I=\frac{\Phi_m}{L}=\frac{\mu_0 m b^2}{2L(b^2+z^2)^{\frac{3}{2}}}.$

(2)

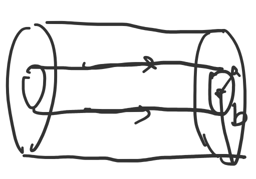


$B=\int\frac{\mu_0}{4\pi}\frac{Id\vec{l}}{r}$

$=\frac{\mu_0 I}{4\pi}\frac{r\,d\theta}{\sqrt{r^2+z^2}}\times 2\pi=\frac{r\mu_0 I}{2\sqrt{r^2+z^2}}\times\frac{z}{\sqrt{r^2+z^2}}$

$W=\vec{m}\cdot\vec{B}=\frac{m r\mu_0 I}{2\sqrt{r^2+z^2}}=\frac{\mu_0^2m^2b^4}{4L(b^2+z^2)^3}.$

5.



$B=\begin{cases}\frac{\mu_0}{2\pi r}\frac{r^2}{b^2}I, & r<b\\ \frac{\mu_0}{2\pi r}I, & r>b.\end{cases}$

$w=\frac{1}{2}\frac{B^2}{\mu_0}=\begin{cases}\frac{1}{2}\mu_0\left(\frac{r}{2\pi b^2}I\right)^2=\frac{\mu_0 r^2I^2}{8\pi^2b^4}\\ \frac{1}{2}\mu_0\left(\frac{1}{2\pi r}I\right)^2=\frac{\mu_0 I^2}{8\pi^2r^2}.\end{cases}$

$W=\int_0^b\cancel{2\pi}\frac{\mu_0 r^2I^2}{\cancel{8\pi^2}b^4}r\,dr+\int_b^{+\infty}\frac{\mu_0 I^2}{8\pi^2r^2}r\,dr\ \cancel{2\pi}$

$=\frac{\mu_0 I^2}{4\pi}\left(\int_0^b\frac{1}{b^4}r^3dr+\int_b^{+\infty}\frac{1}{r}dr\right)$

$=\frac{\mu_0 I^2}{4\pi}\left(b^4/4b^4+\ln\frac{b}{a}\right)=\frac{\mu_0 I^2}{16\pi}+\frac{\mu_0 I^2}{4\pi}\ln\frac{a}{b}$

$L=\frac{2W}{I^2}=\frac{\mu_0}{8\pi}+\frac{\mu_0}{2\pi}\ln\frac{b}{a}.$