

例题


2022年6月22日 星期三 下午10:02

例. 电容 C 带电量 Q.

t=0 时放电, 线圈 L, 

(1) 求 L 内磁场第一次等于 C 内电场时的时刻 t₁.

(2) 求 L 内磁场第二次到达极大值的时刻 t₂.

 $L = \frac{d\Phi}{dt}, L \frac{dI}{dt} + \frac{Q}{C} = 0.$
 $\therefore \frac{d^2 Q}{dt^2} + \omega^2 Q = 0, \omega = \frac{1}{\sqrt{LC}}.$

$Q(t) = Q_0 \cos \omega t, I(t) = -\omega Q_0 \sin \omega t.$

\therefore 电量 $W_e = \frac{Q^2}{2C} = \frac{Q_0^2}{2C} \cos^2 \omega t,$

$W_m = \frac{1}{2} L I^2 = \frac{Q_0^2}{2C} \sin^2 \omega t.$

$\therefore t_1 = \frac{T}{8} = \frac{\pi}{4\omega} = \frac{\sqrt{LC}}{4}, t_2 = \frac{3\sqrt{LC}}{4}.$

例. 无限长螺线管

$H = nI, B = \mu_0 \mu_r n I.$

$\Phi = NBS = \mu_0 \mu_r N n I S = \mu_0 \mu_r n^2 I L S$

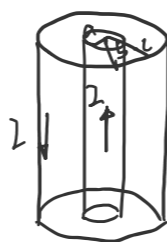
$L = \frac{\Phi}{I} = \mu_0 \mu_r n^2 V.$

$W = \frac{1}{2} L I^2 = \frac{1}{2} \mu_0 \mu_r n^2 I^2 V = \frac{1}{2} B H V$

$W = \frac{1}{2} \iiint_V \vec{B} \cdot \vec{H} dV = \iiint_V w_m dV.$

2. 磁能求电感.

例.



中心半径为 a 的导线, 外部内外径为 b, c 的圆筒, 其间充满 μ_r 的介质, 求单位长度的电感.

$H = \begin{cases} \frac{1}{2\pi r} \frac{r^2}{a^2} I, & 0 < r < a \\ \frac{1}{2\pi r} I, & a < r < b \\ \frac{1}{2\pi r} (1 - \frac{r^2 - b^2}{c^2 - b^2}) I, & b < r < c \\ 0, & r > c \end{cases}$ $\frac{c^2 - r^2}{c^2 - b^2}$

$B = \begin{cases} \frac{\mu_0 I}{2\pi r} \frac{r^2}{a^2}, & 0 < r < a \\ \frac{\mu_0 \mu_r I}{2\pi r}, & a < r < b \\ \frac{\mu_0}{2\pi r} (1 - \frac{r^2 - b^2}{c^2 - b^2}) I, & b < r < c \\ 0, & r > c \end{cases}$

$w = \frac{1}{2} \vec{B} \cdot \vec{H} = \begin{cases} \frac{1}{2} \mu_0 (\frac{r}{2\pi a^2} I)^2, & 0 < r < a \\ \frac{1}{2} \mu_0 \mu_r (\frac{1}{2\pi r} I)^2, & a < r < b \\ \frac{1}{2} \mu_0 [\frac{1}{2\pi r} (1 - \frac{r^2 - b^2}{c^2 - b^2}) I]^2, & b < r < c \\ 0, & r > c \end{cases}$

$W = \int_0^L dz \int_r^r r dr \int_0^{2\pi} w d\theta$ $(\frac{c^2 - r^2}{c^2 - b^2})^2 = \frac{c^4}{(c^2 - b^2)^2} + r^2 - 2c^2$

$= 2\pi L \left[\frac{1}{2} \mu_0 (\frac{I}{2\pi a^2})^2 \times \frac{1}{4} r^4 \Big|_0^a + \frac{1}{2} \mu_0 \mu_r (\frac{I}{2\pi r})^2 \times \ln r \Big|_a^b + \right.$

$\left. \frac{1}{2} \mu_0 \mu_r (\frac{I}{2\pi (c^2 - b^2)})^2 \left(c^4 \ln r \Big|_b^c + \frac{1}{4} r^4 \Big|_b^c - c^2 r^2 \Big|_b^c \right) \right]$

$L = \frac{2W}{I^2} \times \frac{1}{L} = \frac{\mu_0}{2\pi} \left[\frac{1}{4} + \mu_r \ln \frac{b}{a} + \frac{1}{(c^2 - b^2)^2} \left(c^4 \ln \frac{c}{b} - \frac{1}{4} (c^2 - b^2)(3c^2 - b^2) \right) \right]$

计算自成 $\begin{cases} \text{感应电动势} & L = - \frac{\mathcal{E}_i}{\frac{dI}{dt}} \\ \text{磁通量} & L = \frac{\Phi}{I} \\ \text{磁能} & L = \frac{2W_m}{I^2} \end{cases}$