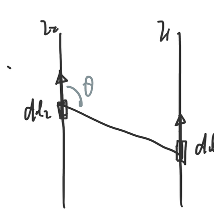
例题

2022年4月24日 星期日　　上午8:21

一、安培定理

例.



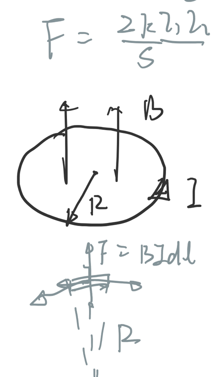
$dF_{12} = kI_1I_2\,dl_1\int_{-\infty}^{\infty}\frac{s\theta}{r_{12}^2}dz$

$z = -s\cot\theta$, $\frac{dz}{r_{12}^2} = \frac{s\,d\theta}{r_{12}^2 s^2\theta} = \frac{d\theta}{s}$

$dF_{12} = \frac{kI_1I_2dl_1}{s}\int_0^{\pi}s\theta\,d\theta = \frac{2kI_1I_2}{s}dl_1$

$F = \frac{2kI_1I_2}{s}l$.

1.



求圆导线内部张力.

$dF = 2BIRd\frac{\theta}{2}$

$2Td\frac{\theta}{2} = 2BIRd\frac{\theta}{2}$

$T = BIR$

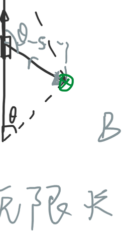
2.

使圆柱体不向下滚动的电流I.

$N2BILR s\theta = mgR s\theta$. $I = \frac{mg}{2NBL}$.

二、毕奥-萨伐尔-拉普拉斯

① 载流直导线.



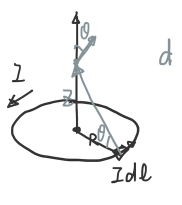
$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Idl\times\hat{r}}{r^2}$　　$dz = d\frac{s}{\tan\theta} = \frac{s}{s^2\theta}d\theta$

$= \frac{\mu_0}{4\pi}\frac{Idz}{r^2} = \frac{\mu_0}{4\pi}\frac{I\frac{s}{s^2\theta}d\theta}{\frac{s^2}{s^2\theta}} = \frac{\mu_0}{4\pi}\frac{Id\theta}{s}$

$B = \int_{\theta_1}^{\theta_2}\frac{\mu_0I}{4\pi s}s\theta\,d\theta$.

无限长, $\vec{B} = \frac{\mu_0I}{4\pi s}\int_0^{\pi}s\theta\,d\theta = \frac{\mu_0I}{2\pi s}\hat{\varphi}$.

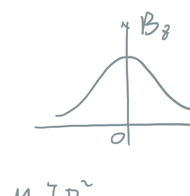
② 载流圆环轴线上.



$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Idl}{R^2+z^2}$,

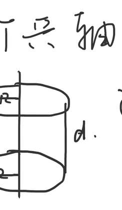
$\vec{B} = \frac{\mu_0}{4\pi}\oint_l\frac{I}{R^2+z^2}\cdot\frac{R}{\sqrt{R^2+z^2}}dl$

$= \frac{\mu_0I\times2\pi R\times R}{4\pi(R^2+z^2)^{\frac{3}{2}}} = \frac{\mu_0IR^2}{2(R^2+z^2)^{\frac{3}{2}}}$.



③ 亥姆霍兹线圈.

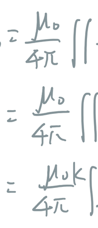
两个共轴同向的线圈磁场叠加.



线圈中间磁场均匀.

④ 无限长螺线管.

单位长度线管匝数为n, 每匝电流为I.



对称性: z轴方向.

面电流密度 $k = nI$.

Idl

$jdsdl$

kdS

$d\vec{B} = \frac{\mu_0}{4\pi}\frac{K\times\hat{R}}{R^2}dS$

$B = \frac{\mu_0}{4\pi}\iint\frac{\vec{K}\times\hat{R}}{R^2}dS$

$= \frac{\mu_0}{4\pi}\iint\frac{K\,dl\,dz}{R^2} = \frac{\mu_0K}{4\pi}\int_{-\infty}^{\infty}dz\oint_C\frac{d\vec{l}\times\vec{R}}{R^3}$

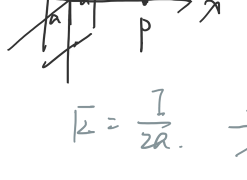
$= \frac{\mu_0K}{4\pi}\int_{-\infty}^{\infty}dz\oint_l\frac{d\vec{l}\times(-z\hat{z}-\vec{R})}{R^3}$

$= \frac{\mu_0K}{4\pi}\oint_{C_0}\vec{R}\times d\vec{R}\int_{-\infty}^{\infty}\frac{dz}{(z^2+R^2)^{\frac{3}{2}}}$

$= \frac{\mu_0K}{2\pi}\oint_{C_0}\frac{\vec{R}\times d\vec{R}}{R^2}$　　$\vec{R}\times d\vec{R} = R^2d\theta\,\hat{z}$.

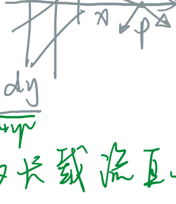
$= \frac{\mu_0K}{2\pi}\oint_{C_0}d\theta = \begin{cases}0, & C_0外\\ (\mu_0K)\hat{z}, & C_0内.\end{cases}$

1.



求P的磁感应强度.

$K = \frac{I}{2a}$.



$d\vec{B} = \frac{\mu_0}{2\pi}\frac{\frac{I}{2a}dy}{\sqrt{x^2+y^2}}$

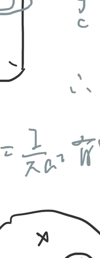
模型: 无限长载流直导线.

$B = \frac{\mu_0\frac{I}{2a}}{2\pi}\int_{-a}^{a}\frac{1}{\sqrt{x^2+y^2}}\cdot\frac{x}{\sqrt{x^2+y^2}}dy$

$= \frac{\mu_0Ix}{4\pi a}\int_{-a}^{a}\frac{dy}{(x^2+y^2)} = \frac{\mu_0I}{2\pi a}\arctan(\frac{a}{x})$

三、静磁场基本定理

例. 圆形直导线截面半径a, 电流I均匀流过导体截面. 求B分布.



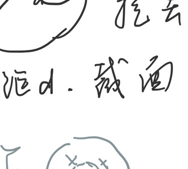
环路定理 → 画个回路.

$\oint_C\vec{B}\cdot d\vec{l} = B\cdot2\pi r = \begin{cases}\mu_0\frac{r^2}{a^2}I, & r<a\\ \mu_0I, & r\geq a\end{cases}$

$\therefore \vec{B} = \begin{cases}\frac{\mu_0rI}{2\pi a^2} = \frac{\mu_0}{2}\vec{j}\times\vec{r}, & r<a\\ \frac{\mu_0I}{2\pi r} = \frac{\mu_0}{2}\frac{a^2}{r^2}\vec{j}\times\vec{r}, & r\geq a\end{cases}$

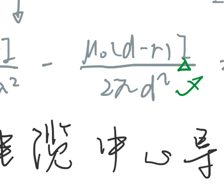
$*\vec{j} = \frac{I}{\pi a^2}$ 方向

例.



半径为a的圆柱直导线中挖去半径b的空管, 两轴线相距d. 截面上流过I, 求空管内B.

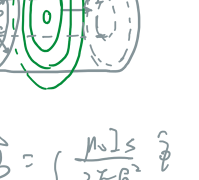
看作



$= \frac{\mu_0}{2}\vec{j}\times\vec{d}$.

$\frac{\mu_0rI}{2\pi a^2} - \frac{\mu_0(d-r)I}{2\pi d^2} = \frac{\mu_0}{2}\vec{j}\times\vec{r} + \frac{\mu_0}{2}\vec{j}(\vec{d}-\vec{r})$

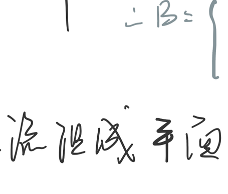
例. 同轴电缆中心导线半径a, 外部是内径b外径c的圆筒, 内外部电流相反流动I, 求各区域B.



$B\cdot2\pi r = \begin{cases}\mu_0\frac{r^2}{a^2}I, & r\leq a\\ \mu_0I & a<r\leq b\\ \mu_0I-\mu_0\frac{r^2-b^2}{c^2-b^2}I, & b<r\leq c\\ 0, & r>c\end{cases}$

$\therefore\vec{B} = \begin{cases}\frac{\mu_0Is}{2\pi a^2}\hat{\varphi} & r\leq a\\ \frac{\mu_0I}{2\pi r}\hat{\varphi} & a<r\leq b\\ \frac{\mu_0I}{2\pi r}\frac{c^2-r^2}{c^2-b^2}\hat{\varphi} & b<r\leq c\\ 0 & r>c\end{cases}$

例. 面电流密度度 $\vec{K} = K\hat{y}$ 的电流通过平面导体板. 求空间的B.

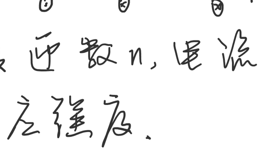


对称性, 仅沿平行平面.

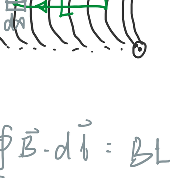
$\oint_C\vec{B}\cdot d\vec{l} = B\cdot2\times L = \mu_0\vec{K}L$

$\therefore\vec{B} = \begin{cases}\frac{1}{2}\mu_0K\hat{x}, & z>0\\ -\frac{1}{2}\mu_0K\hat{x}, & z<0.\end{cases}$　　$\vec{B} = \frac{1}{2}\mu_0\vec{K}\times\hat{n}$.

电流组成平面.

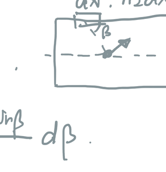


例. 螺线管单位长度匝数n, 电流I, 求其内外的磁感应强度.



对称性, 竖着的被相邻两根电流产生相反的B抵消, 外侧回路取无穷远, 为0.

$\therefore\oint_C\vec{B}\cdot d\vec{l} = BL = \mu_0nLI$, $B = \mu_0nI$

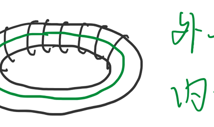


dx内. $Indx$, $dB = \frac{\mu_0R^2Indx}{2(x^2+R^2)^{\frac{3}{2}}}$.

$dx = -\frac{(x^2+R^2)}{R}d\beta$, $\therefore dB = -\frac{\mu_0nI\sin\beta}{2}d\beta$.

$\therefore B = -\frac{1}{2}\mu_0nI\sin\beta\,d\beta = \frac{1}{2}\mu_0nI(\cos\beta_1-\cos\beta_2)$

例. 环形螺线管内外半径a、b, 总匝数N且通有电流I, 求内外的B.

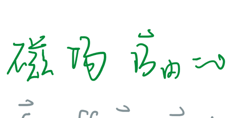


外部: $B=0$.

内部: $B\times2\pi r = \mu_0NI$,

$B = \frac{\mu_0NI}{2\pi r} = \mu_0nI$.

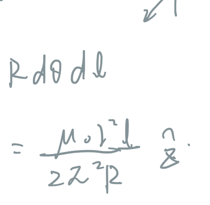
例.



半径为R的无限长圆柱面上沿轴向通有I, 求一段长为l的半圆柱面的受力.

磁场 $\vec{B}_内=0$, $\vec{B}_外 = \frac{\mu_0I}{2\pi r}\hat{\varphi}$

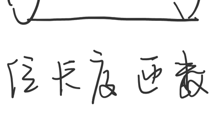
$\vec{f} = \iint_S\vec{K}\times\vec{B}\,dS$



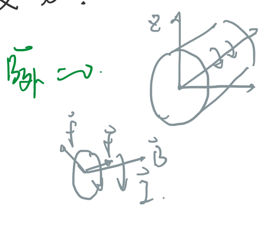
$= \iint_S\frac{I}{2\pi R}\times\frac{\mu_0I}{2\pi R}\cos\theta\,dS$

$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_0^l\frac{\mu_0I^2}{4\pi^2R^2}\times\cos\theta\,R\,d\theta\,dl$

$= 2\times\frac{\mu_0I^2}{4\pi^2R^2}\times R\times l = \frac{\mu_0I^2l}{2\pi^2R}\hat{z}$.



半径为R的无限长圆柱面上密绕螺线管, 单位长度匝数为n, 每匝线圈电流为I, 求半圆柱面上的受力.



磁场 $\vec{B}_内 = \mu_0nI\hat{y}$. $\vec{B}_外=0$.

$\therefore\langle\vec{B}\rangle = \frac{1}{2}\mu_0nI\hat{y}$.

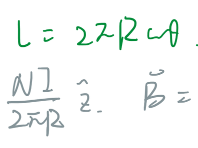
$\vec{F} = \iint_S\vec{K}\times\langle\vec{B}\rangle d\vec{S}$

$= \iint_S nI\times\frac{1}{2}\mu_0nIR\cos\theta\,dS$

$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_0^l\frac{1}{2}\mu_0n^2I^2R\cos\theta\,d\theta\,dl$

$= \mu_0n^2I^2Rl\,\hat{z}$.

一般情况



螺射角θ.

N匝, 求单位面积受力.

经过距离 $L = 2\pi R\cot\theta$, 绕柱一周.

$\theta=0$, $\vec{K} = \frac{NI}{2\pi R}\hat{z}$. $\vec{B} = \begin{cases}0 & 内\\ \frac{\mu_0NI}{2\pi s}\hat{\varphi} & 外\end{cases}$

$\theta=\frac{\pi}{2}$, $\vec{K} = \frac{NI}{L}\hat{\varphi}$, $\vec{B} = \begin{cases}\frac{\mu_0NI}{L}\hat{z} & 内\\ 0 & 外\end{cases}$

则 $\vec{K}(\theta) = \hat{z}\frac{NI}{2\pi R}\cos\theta + \hat{\varphi}\frac{NI}{L}\sin\theta$

$= \frac{NI}{2\pi R\cos\theta}(\hat{z}\cos^2\theta + \hat{\varphi}\sin^2\theta)$

$\vec{B}_内 = \hat{z}\frac{\mu_0NI}{2\pi R\cos\theta}\sin^2\theta$, $\vec{B}_外 = \hat{\varphi}\frac{\mu_0NI}{2\pi r}\cos\theta$.

$\langle\vec{B}\rangle = \frac{\mu_0NI}{4\pi R\cos\theta}(\hat{z}\sin^2\theta + \hat{\varphi}\cos^2\theta)$

$\vec{f} = \vec{K}\times\langle\vec{B}\rangle = \hat{s}\frac{\mu_0N^2I^2}{8\pi^2R^2}(\tan^2\theta-1)$.

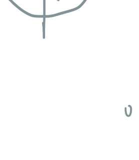
$\theta = \frac{\pi}{4}$, $L = 2\pi R$, $\vec{f}=0$.

$\theta>\frac{\pi}{4}$, 排斥力. $\theta<\frac{\pi}{4}$, 吸引力. (pinch)

* 磁偶极矩.

$\vec{B} = -\frac{\mu_0\vec{m}}{4\pi r^3} + \frac{3\mu_0\vec{r}(\vec{m}\cdot\vec{r})}{4\pi r^5}$

5.6　假定地球的磁场是由地球中心的小电流环产生的，已知地面磁极（电流环轴线与地面的交点）附近磁场为 0.8G，地球半径 $R = 6\times10^6$m，求小电流环的磁矩.



$\vec{B} = \frac{-\mu_0r^2\vec{m}+3\mu_0mr\vec{r}}{4\pi r^5}$

$= \frac{-\mu_0\vec{m}+3\mu_0m\vec{e}_r}{4\pi r^3} = \frac{-\mu_0m+3\mu_0m}{4\pi r^3}\vec{e}_r$

$0.8 = \frac{2\times\mu_0\times m}{4\pi\times216\times10^{18}}$, $m=$