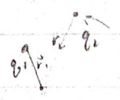
知识点

2022年2月9日 10:24

静电能.

一、真空中点电荷间的相互作用能.



$U_{12} = -\int_{\infty}^{2} \vec{E}\cdot d\vec{l} = -\int_{\infty}^{2} \frac{q_1}{4\pi\varepsilon_0 r_{12}^2} dl = \frac{q_1}{4\pi\varepsilon_0 r_{12}}$，$W_{12} = \frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}} = q_2 U_{12}$

$W_{12} = W_{21} = \frac{1}{2}(q_1 U_{21} + q_2 U_{12})$

$U_i = \sum_{j=1, j\neq i}^{N} U_{ji} = \frac{1}{4\pi\varepsilon_0} \sum \frac{q_j}{r_{ij}}$，$W_互 = \frac{1}{2}\sum_{i=1}^{N} q_i U_i = \frac{1}{8\pi\varepsilon_0} \sum_{i,j=1, i\neq j}^{N} \frac{q_i q_j}{r_{ij}}$

二、连续电荷分布的静电能

体电荷 $W_e = \frac{1}{2}\iiint_V \rho_e(\vec{r}) U_1(\vec{r}) dV = \frac{1}{2}\iiint_V \rho_e(\vec{r}) U(\vec{r}) dV$.

面电荷 $W_e = \frac{1}{2}\iint_S \sigma_e(\vec{r}) U(\vec{r}) dS$.

多个带电体组成的系统：$U(\vec{r}) = U_i(\vec{r}) + U_自(\vec{r})$，$W_e = W_互 + W_自$.

$W_互 = \sum_{i=1}^{N} \frac{1}{2}\iiint_{V_i} \rho_e(\vec{r}) U_i(\vec{r}) dV$. $W_自 = \sum_{i=1}^{N} \iiint_{V_i} \rho_e(\vec{r}) U_自(\vec{r}) dV$.

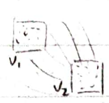
$\Rightarrow W_互 = \frac{1}{2}\sum_{i=1}^{N} q_i U_i = \frac{1}{2}\sum_{i=1}^{N} \int_{L_i} \lambda_e(l) U_i(l) dl$.

空间中有电介质（线性无损耗），$\rho_e(\vec{r}) = \rho_{自由}(\vec{r}) + \rho_{极化}(\vec{r})$.

系统宏观静电能 $W_e = \frac{1}{2}\iiint_{V_0} \rho_{e0}(\vec{r}) U(\vec{r}) dV + \frac{1}{2}\iiint_{V_0} \rho_e'(\vec{r}) U(\vec{r}) dV$.

外界对系统做功 $W = W_e + W_极$，→ 系统的静电能.

三、电荷体系在外电场中的静电能.



$W_互 = \frac{1}{2}\iiint_{V_1} \rho_{e1}(\vec{r}_1) U_1(\vec{r}_1) dV_1 + \frac{1}{2}\iiint_{V_2} \rho_{e2}(\vec{r}_2) U_2(\vec{r}_2) dV_2 \longrightarrow \iiint_V \rho_e(\vec{r}) U(\vec{r}) dV$

$U_1(\vec{r}_1) = \frac{1}{4\pi\varepsilon_0}\iiint_{V_2} \frac{\rho_{e2}(\vec{r}_2)}{|\vec{r}_1 - \vec{r}_2|} dV_2$，$U_2(\vec{r}_2) = \frac{1}{4\pi\varepsilon_0}\iiint_{V_1} \frac{\rho_{e1}(\vec{r}_1)}{|\vec{r}_2 - \vec{r}_1|} dV_1$.

电荷体系与外静电场的相互作用能 → 静电能 $W_e = \iiint \rho_e(\vec{r}) U(\vec{r}) dV$.

四、电场的能量和能量密度.

$W_e = \frac{1}{2}QU$. $Q = \sigma_e S = DS$，$U = Ed$，$W_e = \frac{1}{2}DSEd = \frac{1}{2}DEV$.

电能密度（单位体积的静电能）$w_e = \frac{W}{V} = \frac{1}{2}\vec{D}\cdot\vec{E}$. $W = \iiint_V w_e dV = \frac{1}{2}\iiint_V \vec{D}\cdot\vec{E} dV$.

$\vec{D} = \varepsilon_0\vec{E} + \vec{P}$，$W = \frac{1}{2}\iiint_V \varepsilon_0 \vec{E}^2 dV + \frac{1}{2}\iiint \vec{P}\cdot\vec{E} dV = W_{e0} + W_极$.

五、非线性介质及电滞损耗

外界做功 $dA = U dq = El\, dD_n S = E\, dD_n Sl = \vec{E}\cdot d\vec{D}\, V$，单位体积 $da = \frac{dA}{V} = \vec{E}\cdot d\vec{D}$.

$\therefore da' = d(\frac{\varepsilon_0}{2}E^2) + \vec{E}\cdot d\vec{P}$. 宏观静电能 + 极化功（线性无损、非线性有损）

无损：极化功 → 极化能，$da' = dw_e$.

有损：电滞损耗 $a' = \oint a' = \oint E dP$，电滞回线所围面积.

六、利用静电能求静电力.

静电力 $\vec{F}$ 做功 $\delta A = \vec{F}\cdot\delta\vec{r}$.

系统孤立：$\delta W_e = -\delta A$，$\vec{F} = -(\nabla W_e)_Q$.（系统Q恒定）.

系统非孤立：$\delta W_e = -\delta A + \delta A'$，外界做功.

设系统U恒定，$dA' = \sum_{i=1}^{N} U_i \delta Q_i$（系统内各导体）

$\delta W_e = \frac{1}{2}\sum_{i=1}^{N} U_i \delta Q_i$，$\therefore \delta A' = 2\delta W_e$. $\therefore \delta W_e = \delta A$. $\vec{F} = \nabla W_e$.

一般情况：设想系统状态变化（虚位移 $\delta\vec{r}$，虚功 δA）