课后题

2022年3月31日 星期四　　上午7:07

1. $V=\frac{q}{4\pi\varepsilon_0 a}$

$W=\frac{1}{2}\times 2\times\frac{q^2}{4\pi\varepsilon_0 a}=\frac{1}{2}\times\frac{9\times10^9\times1.6\times10^{-19}\times1.6\times10^{-19}}{6\times10^{-15}}$

$=3\times0.64\times10^{-14}=1.92\times10^{-14}$ J.　3.8×10^{-14} J

$F=\frac{1}{4\pi\varepsilon_0}\frac{q^2}{a^2}=\frac{9\times10^9\times2.56\times10^{-38}}{36\times10^{-30}}=6.4$ N.

2. (1) $W_1=W_2=q\times\frac{2q}{4\pi\varepsilon_0 a^2}=\frac{q^2}{2\pi\varepsilon_0 a^2}$.

$W_3=\frac{q^2}{4\pi\varepsilon_0(2a)^2}=\frac{q^2}{8\pi\varepsilon_0 a^2}$.

(2) $W=W_1+W_2+W_3=\frac{7}{8}\frac{q^2}{\pi\varepsilon_0 a}$

3. 略.

4. (1)  $E=\frac{e}{4\pi\varepsilon_0 r^2}$, $U=\int_r^\infty\frac{e}{4\pi\varepsilon_0 r^2}dr=\frac{e}{4\pi\varepsilon_0 r}$.

$W=eU=\frac{e^2}{4\pi\varepsilon_0 r}=mc^2$,

$r=\frac{4\pi\varepsilon_0 mc^2}{e^2}$

S1. $E=\frac{\rho_0}{3\varepsilon_0}r$. $U=\int\vec{E}\,d\vec{l}$

$W=\frac{1}{2}\int_0^R U\,dq$.

S2. $w=\frac{1}{2}\varepsilon_0 E^2$, $W=\int w\,dV$.

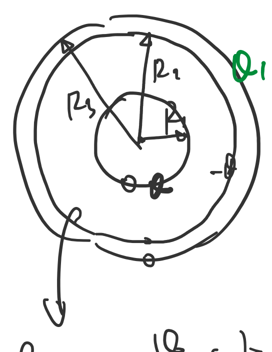
S3. C.

(2)  $W=\int_0^r\frac{1}{4\pi\varepsilon_0 r^2}\times\frac{e}{\frac{4}{3}\pi r^3}\times4\pi r^2\,dr$?

$\frac{1}{4\pi\varepsilon_0 r}\times\frac{r^3}{a^3}e$　　$\frac{3}{20}\frac{Q^2}{2\varepsilon_0 V}=mc^2$.

$r=\frac{20\pi\varepsilon_0 mc^2}{3e^2}$

(3) $r_0=\frac{1.6\times1.6\times10^{-38}\times9}{9.11\times10^{-31}\times9\times10^{16}}\times10^{23}=\frac{2.56}{9.11}\times10^{-23}$.

5. 

$E=\begin{cases}0, & r\le R_1\\ \frac{Q}{4\pi\varepsilon_0 r^2}, & R_1\le r\le R_2\\ 0, & R_2\le r\le R_3\\ \frac{Q}{4\pi\varepsilon_0 r^2}, & r\ge R_3\end{cases}$

(1) $\varphi_1=\frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R_3}+\frac{1}{R_1}-\frac{1}{R_2}\right)$

$\varphi_2=\int_{R_3}^\infty\frac{Q}{4\pi\varepsilon_0 r^2}dr=\frac{Q}{4\pi\varepsilon_0 R_3}$,

$W=\frac{1}{2}\frac{Q^2}{4\pi\varepsilon_0}\left(\frac{1}{R_3}+\frac{1}{R_1}-\frac{1}{R_2}\right)$

(2) 3 导体. $\varphi=\frac{Q}{4\pi\varepsilon_0 R_3}$, $W=\frac{1}{2}\frac{Q^2}{4\pi\varepsilon_0 R_3}$

S1. E U W

S2. W

S3 $C=\frac{4\pi\varepsilon_0 R_1R_2}{R_2-R_1}$

$W=\frac{1}{2C}Q^2$

C_1. C_2.

6. 

$E=\begin{cases}\frac{Q}{4\pi r^2\varepsilon_0}, & r\ge R\\ \frac{1}{4\pi r^2\varepsilon_0}\times\frac{r^3}{R^3}Q, & r<R. =\frac{rQ}{4\pi\varepsilon_0 R^3}\end{cases}$

$\varphi=\begin{cases}\frac{Q}{4\pi\varepsilon_0 r}, & r\ge R\\ \frac{Q}{4\pi\varepsilon_0 R}+\frac{1}{2}\times\frac{r^2Q}{4\pi\varepsilon_0 R^3}, & r<R\end{cases}$

$=\int_R^\infty\frac{Q}{4\pi\varepsilon_0 r^2}+\int_r^R\frac{rQ}{4\pi\varepsilon_0 R^3}dr$

$=\frac{Q}{4\pi\varepsilon_0 R}+\frac{Q(R^2-r^2)}{2\times4\pi\varepsilon_0 R^3}$, $r<R$.

$U=\frac{1}{2}\int_0^R\frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R}+\frac{R^2-r^2}{2R^3}\right)\times\frac{Q}{\frac{4}{3}\pi R^3}\times4\pi r^2\,dr$

$=\frac{1}{2}\int_0^R\frac{3Q^2}{4\pi\varepsilon_0 R^3}\times\frac{3R^2-r^2}{2R^3}\times r^2\,dr$

$=\frac{1}{2}\frac{3Q^2}{8\pi\varepsilon_0 R^6}\int_0^R(3R^2\times r^2-r^4)\,dr$

$=\frac{1}{2}\frac{3Q^2}{8\pi\varepsilon_0 R^6}\left(R^2\times R^3-\frac{1}{5}R^5\right)=\frac{4}{5}\times\frac{3Q^2}{8\pi\varepsilon_0 R}=\frac{3Q^2}{20\pi\varepsilon_0 R}$.

$U=\frac{3}{5}\times\frac{Q^2}{4\pi\varepsilon_0 R}=\frac{3}{5}\times9\times\frac{1.6\times1.6\times10^{-38}\times92\times92}{9.2\times10^{-15}}$

$=0.6\times2.56\times10^{-23}\times9\times92\times10\times10^{?}$

(2) 裂成 4be. R?

$U=\frac{3}{5}\times9\times10^9\times\frac{1.6\times1.6\times10^{-38}\times46\times46}{9.2\times10^{-15}}$

$2\times\frac{4\pi}{3}r_1^3=\frac{4\pi}{3}r^3$ → 总体积不变.

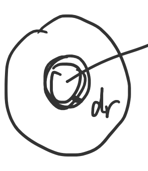
[illegible]

6. (right column) 

$E=\begin{cases}\frac{1}{4\pi r^2}\times\frac{r^3}{R^3}Q, & 0<r<R =\frac{r}{4\pi R^3}Q.\\ \frac{1}{4\pi r^2}Q, & r>R.\end{cases}$

92e.

$\rho=\frac{Q}{\frac{4}{3}\pi R^3}$



$q=\frac{r^3}{R^3}Q$.　dr: $dq=\rho\,4\pi r^2dr$

$E=\frac{\frac{r^3}{R^3}Q}{4\pi r^2}=\frac{rQ}{4\pi R^3}$.　$\varphi=\frac{r^2Q}{8\pi R^3}$.

$\int_0^R\varphi\times\rho\,4\pi r^2dr$

$=\int_0^R\frac{r^2Q}{8\pi R^3}\times\frac{Q}{\frac{4}{3}\pi R^3}\times4\pi r^2\,dr$

$=\int_0^R\frac{3Q^2r^4}{8\pi R^6}dr=\frac{3Q^2}{20\pi R\varepsilon_0}$

$W=\frac{3}{20}\times\frac{9\times92\times92\times1.6\times1.6\times10^{-38}}{3.14\times9.2\times10^{-15}\times8.85\times10^{-12}}$

$=\frac{46\times2.56}{8.85}\times\frac{3}{3.14}\times10^{-11}=$

(2) 体积不变. q 均分. $r_1=r_2$　$r_1^3=\frac{R^3}{2}$

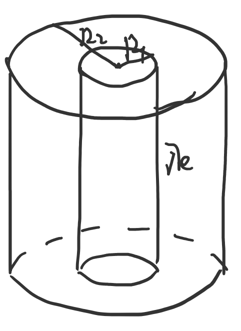
$\frac{4}{3}\pi r^3=\frac{4}{3}\pi r_1^3+\frac{4}{3}\pi r_2^3$, $r^3=r_1^3+r_2^3$.

$\Delta W=\frac{3}{20}\times\frac{Q^2}{\pi R\varepsilon_0}-\frac{3}{20}\frac{(\frac{1}{2}Q)^2}{\pi\frac{1}{\sqrt[3]{2}}R\varepsilon_0}\times2$

$=\frac{3}{20}\frac{Q^2}{\pi R\varepsilon_0}\left(1-\sqrt[3]{2}\times\frac{1}{4}\times2\right)$

$=\frac{3}{20}\frac{Q^2}{\pi R\varepsilon_0}\left(1-\frac{\sqrt[3]{2}}{2}\right)$

(3). 1kg　$\frac{1}{92\times1.66\times10^{-27}}\times\Delta W$

9. 

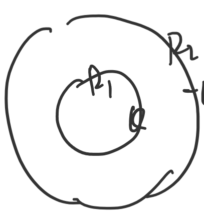
$E\times2\pi rl=\frac{\lambda_e l}{\varepsilon_0}$, $E=\frac{\lambda_e}{2\pi R\varepsilon_0}\le E_b$.　$R_1=\frac{\lambda_e}{2\pi\varepsilon_0 E_b}$

$V=\int_{R_1}^{R_2}\vec{E}\cdot d\vec{l}=\int_{R_1}^{R_2}\frac{\lambda_e}{2\pi\varepsilon_0}\cdot\frac{1}{r}dr$

$=\frac{\lambda_e}{2\pi\varepsilon_0}\ln\frac{R_2}{R_1}$　$\le E_bR_1\ln\frac{R_2}{R_1}$

$U=\frac{\lambda_e}{2\pi\varepsilon_0}\ln\frac{R_2}{R_1}\times\lambda_e l=\frac{\lambda_e^2 l}{2\pi\varepsilon_0}\ln\frac{R_2}{R_1}$

$V'(R_1)=\frac{\lambda_e}{2\pi\varepsilon_0}\frac{R_1}{R_2}\times\left(-\frac{R_2}{R_1^2}\right)=-\frac{\lambda_e}{2\pi\varepsilon_0 R_1}$　$\le2\pi\varepsilon_0E_b^2R_1^2\ln\frac{R_2}{R_1}$

10. 

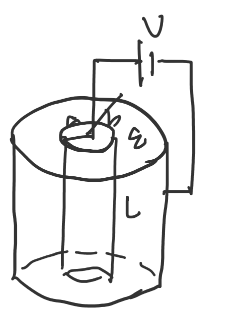
$E=\begin{cases}\frac{Q}{4\pi\varepsilon_0 r^2}, & R_1\le r\le R_2\\ 0, & 0<r<R_1,\ r>R_2.\end{cases}$

$V=\int_{R_2}^\infty0\,dr+\int_{R_1}^{R_2}\frac{Q}{4\pi\varepsilon_0 r^2}dr+\int_0^{R_1}0\,dr$

$=\frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$.

所求 $W_1=\frac{Q^2}{8\pi\varepsilon_0}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

孤立 $W_2=\frac{Q^2}{8\pi\varepsilon_0 R_1}$?

11. 

V 不变.　　$Q=CV$

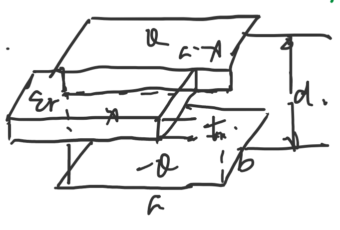
$D=\frac{Q}{2\pi rL}$, $E=\frac{Q}{2\pi r\varepsilon L}$. $U=\frac{Q}{2\pi\varepsilon L}\ln\frac{b}{a}$

$C=\frac{2\pi\varepsilon L}{\ln(\frac{b}{a})}$ → $C'=\frac{2\pi\varepsilon(L-x)}{\ln(\frac{b}{a})}+\frac{2\pi\varepsilon_0 x}{\ln(\frac{b}{a})}$

$W=\frac{Q^2}{2C}=\frac{C^2V^2}{2C}=\frac{1}{2}CV^2$.

$W=\frac{V^2}{2}\times\frac{2\pi\varepsilon L}{\ln(\frac{b}{a})}$, $W'=\frac{V^2}{2}\times\frac{2\pi\varepsilon(L-x)}{\ln(\frac{b}{a})}$.

$F=\frac{\partial W}{\partial x}=\frac{V^2\pi\varepsilon}{\ln(\frac{b}{a})}$　$-\frac{2(\varepsilon-\varepsilon_0)V^2}{\ln(\frac{b}{a})}$

12. 

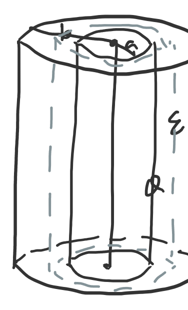
$E=\frac{\sigma S}{S\varepsilon_0}=\frac{\sigma}{\varepsilon_0}$, $V=\frac{\sigma d}{\varepsilon_0}$

$C=\frac{Q}{V}=\frac{\sigma ab}{\frac{\sigma d}{\varepsilon_0}}=\frac{ab\varepsilon_0}{d}$.

$C=\frac{(a-x)b\varepsilon_0}{d-t}+\frac{xb\varepsilon_r}{t}$

$W=\frac{Q^2}{2C}=\frac{Q^2}{2}\times\frac{1}{\frac{(a-x)b\varepsilon_0}{d-t}+\frac{xb\varepsilon_r}{t}}$

$\frac{\partial W}{\partial x}=\frac{Q^2}{2}\times\left(-\frac{1}{\left(\frac{(a-x)b\varepsilon_0}{d-t}+\frac{xb\varepsilon_r}{t}\right)^2}\right)\times\left(-\frac{b\varepsilon_0}{d-t}+\frac{b\varepsilon_r}{t}\right)$

7. 

$E=\begin{cases}\frac{Q}{2\pi rl\varepsilon_0}, & a\le r\le b\\ 0, & r>b.\end{cases}$　$\varphi=\begin{cases}\frac{Q}{2\pi\varepsilon_0 l}\ln\frac{b}{r}, & a\le r\le b\\ 0 & r>b\end{cases}$

(1) $D=\begin{cases}\frac{Q}{2\pi rl}, & a\le r\le b\\ 0, & r>b\end{cases}$　$E=\begin{cases}\frac{Q}{2\pi rl\varepsilon}, & a\le r\le b\\ 0, & r>b.\end{cases}$

$\varphi=\begin{cases}\frac{Q}{2\pi\varepsilon l}\ln\frac{b}{r}, & a\le r\le b.\\ 0 & r>b.\end{cases}$

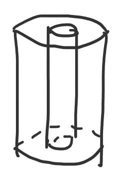
$U=\frac{1}{2}Q\varphi=\frac{Q^2}{4\pi\varepsilon l}\ln\frac{b}{a}=\frac{Q^2}{\pi r^2l^2}\frac{r^2}{4\varepsilon}\ln\frac{b}{r}$

$=\frac{1}{V}\times\frac{(Qr)^2}{4\varepsilon}\ln\frac{b}{a}$.　$\rho=\frac{(Qr)^2}{4\varepsilon}\ln\frac{b}{a}$.

(2) $U=\frac{Q^2}{4\pi\varepsilon l}\ln\frac{b}{a}$.　$w=\frac{1}{2}\vec{D}\cdot\vec{E}=\frac{1}{2}DE$.

(3) $C=\frac{Q}{\varphi}=\frac{2\pi\varepsilon l}{\ln\frac{b}{a}}$

$\frac{Q^2}{2C}=\frac{Q^2}{2\frac{Q}{\varphi}}=\frac{Q\varphi}{2}=\frac{1}{2}Q\frac{Q}{2\pi\varepsilon l}\ln\frac{b}{a}=\frac{Q^2}{4\pi\varepsilon l}\ln\frac{b}{a}=W$.

8. 

$E=\frac{Q}{2\pi rl\varepsilon_0}$

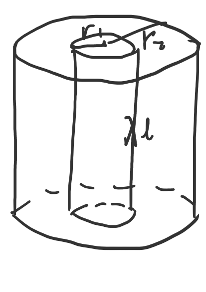
$\varphi=\frac{Q}{2\pi\varepsilon_0 l}\ln\frac{b}{r}$, $\varphi_c=\frac{Q}{2\pi\varepsilon_0 l}\ln\frac{b}{a}$.

$W=\frac{1}{2}Q\varphi=\frac{1}{2}\times\frac{Q^2}{2\pi l\varepsilon_0}\ln\frac{b}{a}=\frac{Q^2}{4\pi\varepsilon_0 l}\ln\frac{b}{a}$.

半径为 $r=\sqrt{ab}$ 的圆柱体内:

$\varphi'=\frac{Q}{2\pi\varepsilon_0 l}\ln\frac{b}{\sqrt{ab}}=\frac{Q}{2\pi\varepsilon_0 l}\ln\sqrt{\frac{b}{a}}$

$W'=\frac{1}{2}Q\varphi'=\frac{1}{2}\times\frac{Q^2}{2\pi l\varepsilon_0}\ln\sqrt{\frac{b}{a}}=\frac{1}{2}W$.

9. 

$E=\frac{\lambda_e l}{2\pi r\varepsilon_0 l}=\frac{\lambda_e}{2\pi\varepsilon_0 r}$　$R_1\le r\le R_2$

$\varphi=\frac{\lambda_e}{2\pi\varepsilon_0}\ln\frac{R_2}{R_1}$.　$W=\frac{1}{2}\frac{\lambda_e^2 l}{2\pi\varepsilon_0}\ln\frac{R_2}{R_1}$

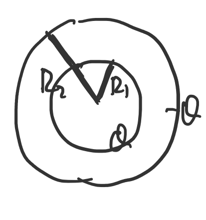
$E_{max}=E_b$, r↓, E↑. $\frac{\lambda_e}{2\pi\varepsilon_0 r}<E_b$, $r>\frac{\lambda_e}{2\pi\varepsilon_0 E_b}$.

(1) R_1↓, φ↑, $\varphi\le\frac{\lambda_e}{2\pi\varepsilon_0}\ln\frac{R_2 2\pi\varepsilon_0 E_b}{\lambda_e}$.

(2) R_1↓, W↑, $W\le\frac{1}{2}\frac{\lambda_e^2 l}{2\pi\varepsilon_0}\ln\frac{2\pi\varepsilon_0 E_b R_2}{\lambda_e}$

FiX.

在计算中忽略 R_1. $U\le E_bR_1\ln\frac{R_2}{R_1}$　$W\le\pi\varepsilon_0E_b^2R_1^2\ln\frac{R_2}{R_1}$.

10. 

$E=\begin{cases}\frac{Q}{4\pi\varepsilon_0 r^2}, & R_1\le r\le R_2\\ 0, & r>R_1\end{cases}$

$\varphi=\begin{cases}\frac{Q}{4\pi\varepsilon r}, & R_1\le r\le R_2.\\ 0 & r>R_1.\end{cases}$

(1) $W_1=\frac{Q^2}{4\pi\varepsilon R_1}$,　$W_2=\frac{Q^2}{4\pi\varepsilon R_1}$

(2)