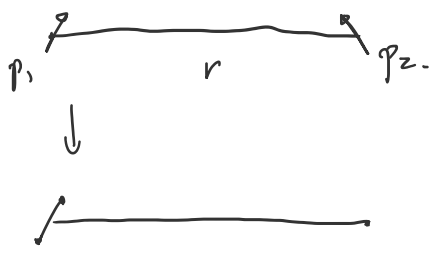
补充题

2022年4月3日 星期日　　下午4:30

1. 点电荷系统静电能 ⇒ 各电荷相互作用能总和.

$W = \frac{1}{2}\sum q_i U_i$

6.



p_2 在 p_1 场中的电势能 $W = -\vec{p}_2 \cdot \vec{E}_{21}$

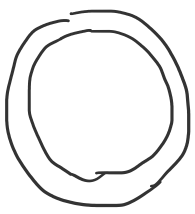
$\vec{E}_{21} = \frac{1}{4\pi\varepsilon_0 r^3}[3(\vec{p}_1\cdot\vec{r})\vec{r} - r^2\vec{p}_1]$

11. $\vec{E} = \frac{Q}{4\pi\varepsilon_0 R^2}$.　$\varphi_R = \frac{Q}{4\pi\varepsilon_0 R} = 900\,V$

$E = \frac{\varphi}{R} = \frac{900\,V}{0.05m} = 1.8\times10^5\ V/m$.

$D = E\varepsilon_0$.　$W = \frac{1}{2}\int D\cdot E = \frac{1}{2}\varepsilon_0 E^2$

15.



$E = \frac{Q}{4\pi\varepsilon_0 R^2}$

球外

$W_0 = \frac{\varepsilon_0}{2}\iiint_V E^2 dV = \frac{\varepsilon_0}{2}\int_R^\infty \left(\frac{Q}{4\pi\varepsilon_0 r^2}\right)^2 \cdot 4\pi r^2 dr$.

若球内场强不为零（有电荷）

$W_1 = \frac{\varepsilon_0}{2}\iiint_V E^2 dV = \frac{\varepsilon_0}{2}\int_0^R E^2 4\pi r^2 dr = 2\pi\varepsilon_0\int_0^R E^2 r^2 dr$.

无电荷时 $W_1 \to 0$

18.

$D = \begin{cases} \frac{1}{4\pi r^3}\times\frac{r^3}{R^3}Q, & 0<r<R. \quad \frac{r}{4\pi R^3}Q \\ \frac{Q}{4\pi r^2}, & r>R. \end{cases}$

$E = \begin{cases} \frac{r}{4\pi\varepsilon_0 R^3}Q, & 0<r<R \\ \frac{Q}{4\pi\varepsilon_0 r^2}, & r>R. \end{cases}$

$U = \begin{cases} \frac{r^2 Q}{8\pi\varepsilon_0 R} - \frac{Q}{4\pi\varepsilon_0 r}, & r>R. \end{cases}$

球外.

$W = \frac{1}{2}\iiint_V \vec{E}\cdot\vec{D}\,dV$

$= \frac{1}{2}\int_R^\infty \frac{Q}{4\pi\varepsilon}\frac{\vec{r}}{r^3}\cdot\frac{Q}{4\pi}\frac{\vec{r}}{r^3} 4\pi r^2 dr$

$= \frac{Q^2}{8\pi\varepsilon}\int_R^\infty \frac{dr}{r^2}$

19.

$E = \begin{cases} 0, & 0\le r\le a,\ b\le r\le d. \\ \frac{Q}{4\pi\varepsilon_0 r}, & a\le r\le b,\ r\ge d \end{cases}$

$U = \begin{cases} \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{d}+\frac{1}{b}-\frac{1}{a}\right), & 0\le r\le a. \\ \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{d}+\frac{1}{r}-\frac{1}{r}\right), & a\le r\le b \\ \frac{Q}{4\pi\varepsilon_0 d}, & b\le r\le d. \\ \frac{U}{4\pi\varepsilon_0 r}, & r\ge d. \end{cases}$

$W_{总} = W_{外1} + W_{外2} = \frac{1}{2}\frac{Q^2}{4\pi\varepsilon_0 a^2} + \frac{1}{2}\frac{Q^2}{4\pi\varepsilon_0 a^2}$

$W_2 = \frac{Q^2}{4\pi\varepsilon_0}\left(\frac{1}{d}-\frac{1}{b}-\frac{1}{c}\right)$?

$W = \frac{\varepsilon_0}{2}\iiint_V E^2 dV = \frac{\varepsilon}{2}\left(\frac{Q}{4\pi\varepsilon_0}\right)^2\left[\int_A^b + \int_c^\infty\right]\frac{1}{r^2}\cdot 4\pi r^2 dr$.

$= \frac{Q^2}{8\pi\varepsilon}\left(\frac{1}{c}-\frac{1}{b}+\frac{1}{d}\right)$