课后题

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1. $E_{11}=\frac{\sigma_1}{2\varepsilon_0}$, $E_{12}=\frac{\sigma_2}{2\varepsilon_0}$, $E_{21}=\frac{\sigma_3}{2\varepsilon_0}$, $E_{22}=\frac{\sigma_4}{2\varepsilon_0}$

板内电场强度为0. $E_{11}=E_{12}+E_{21}+E_{22}$, $E_{22}=E_{11}+E_{12}+E_{21}$

2. $\vec{E}_1\times 4\pi R_1^2=\frac{q}{\varepsilon_0}$, $\vec{E}_1=\frac{q}{4\pi R_1^2\varepsilon_0}$

$\int_{R_1}^{\infty}\frac{q}{4\pi r^2\varepsilon_0}dr=\frac{q}{4\pi\varepsilon_0}\cdot -\frac{1}{r}\Big|_{R_1}^{\infty}=\frac{q}{4\pi\varepsilon_0R_1}$

$dQ=\frac{Q}{\frac{4}{3}\pi(R_3^3-R_2^3)}d(R_3-R_2)\frac{R_3+R_2}{2}\theta$

$=\frac{3Q(R_3+R_2)}{2\pi(R_3^2+R_2^2+R_2R_3)}d\theta$

$dU=\frac{1}{4\pi\varepsilon_0}\frac{dQ}{R}=\frac{3Q\,d\theta}{4\pi^2(R_3^2+R_2^2+R_2R_3)\varepsilon_0}$

$U_{21}=\frac{3Q(R_2+R_3)}{4(R_3^2-R_2^2+R_2R_3)\pi\varepsilon_0}$

$\frac{q}{4\pi\varepsilon_0R_1}+\frac{3Q}{4\pi\varepsilon_0(R_3^2+R_2^2+R_2R_3)}=0$, $q=\frac{3QR_1(R_2+R_3)}{R_3^2+R_2^2+R_2R_3}$

$U_{12}+U_{22}$. $U_{12}=\frac{q}{4\pi\varepsilon_0R_2}=\frac{3QR_1(R_2+R_3)}{4\pi\varepsilon_0(R_2^2+R_3^2+R_2R_3)R_2}$

$U_{22}=$ ...

3. $p\Delta S+2F\sin\frac{\theta}{2}=p\Delta S+2\alpha r\theta^2$ $=pr2\theta^2+2\alpha r\theta^2$

$2\alpha dl$ $=2\alpha r d\frac{\theta}{2}\to\alpha r\theta$

$F=2\alpha\times 2\pi r=4\alpha\pi r$ $4\alpha=p$

$F_{总}=\int p\,dS\cos\theta$ $=\int_0^{2\pi}\int_0^{\frac{\pi}{2}}pr\,d\theta\,d\varphi\cos\theta$ $=2\pi pr\int_0^{\frac{\pi}{2}}\cos\theta\,d\theta=2\pi pr$

4. $U_0>0$.

5. $E_1 4\pi R_1^2=\frac{q_1}{\varepsilon_0}$, $E_2 4\pi R_2^2=\frac{q_2}{\varepsilon_0}$, $0\times 4\pi R_2^2=\frac{q_2+\sigma_2\times 4\pi R_2^2}{\varepsilon_0}$

$0\times 4\pi R_1^2=\frac{q_1+\sigma_1\times 4\pi R_1^2}{\varepsilon_0}$, $\sigma_1=-\frac{q_1}{4\pi R_1^2}$, $\sigma_2=-\frac{q_2}{4\pi R_2^2}$

$F_1=\frac{q_1q}{4\pi r_1^3}\vec{r}_1+\frac{q_1q}{4\pi(r_1+r)^3}(\vec{r}_1+\vec{r})$

6. $C_1=\frac{Q}{V}=\frac{\sigma S}{\int_{d_1}^{d_2}\frac{\sigma}{\varepsilon}\cdot dl}=\frac{\alpha S}{d\frac{\sigma}{\varepsilon_0}}>\frac{\varepsilon_0S}{d}$ ✗

$=\frac{S}{3d}=\frac{...}{3\times0.1}\times10^{-3}=2F$.

$\frac{1}{C}=\frac{1}{C_1}+\frac{1}{C_2}=1$, $C=1F$.

$Q=C_1V_1+C_2V_2=CV=1\times220V=40S$.

7. $C_0=\frac{Q}{V}=\frac{\varepsilon_0S}{d}$

$C_1=\frac{\varepsilon_0S}{d_2}$, $C_2=C_1=\frac{\varepsilon_0\frac{S}{2}}{\frac{d-t}{2}}=\frac{\varepsilon_0S}{d-t}$

$\frac{\varepsilon_0S}{2d}+\frac{\varepsilon_0S}{2(d-t)}$

8. 略.

9. 略.

10. $0.398kg\cdot m^{-3}$.

取$1m^3$. 有$0.398kg$水. $\frac{0.398}{18\times1.66\times10^{-27}}=$ ___ n个水分子.

$n\times6.2\times10^{-30}$

11. (1) $E_{max}=1\times10^9V/m$.

$\vec{E}=\frac{\sigma}{\varepsilon_0}=1\times10^9$. $\sigma=\varepsilon_0\times10^9$ ↓ 8.85×10^{-12}

(2) 在 ΔS内. $n=\frac{\Delta S}{D^2}$个.

$\frac{n\times\Delta l\cdot q}{\Delta S}=\sigma=\varepsilon_0\times10^9$

$\therefore\frac{\frac{\Delta S}{D^2}\Delta l\cdot q}{\Delta S}=\frac{\Delta l\cdot q}{D^2}=\varepsilon_0\times10^9$

$\Delta l=8.85\times10^{-3}\times(2\times10^{-10})^2\times\frac{1}{1.6\times10^{-19}}=$ ___

12. $C_0=\frac{Q}{V}=\frac{Q}{Ed}=\frac{\sigma S}{\frac{\sigma}{\varepsilon_0}d}=\frac{\varepsilon_0S}{d}=\frac{\varepsilon_0\times2}{5\times10^{-3}}=4\times10^2\varepsilon_0$

$C_1=\frac{\varepsilon_1S}{d_1}=\frac{5\times2}{2\times10^{-3}}=5\times10^3F$

$C_2=\frac{\varepsilon_2S}{d_2}=\frac{2\times2}{3\times10^{-3}}=\frac{4}{3}\times10^3F$

$\vec{D}=\varepsilon_0\vec{E}+\vec{P}$

$Q=CU=\frac{\varepsilon_0S}{d}U=4\varepsilon_0\times10^5$ 仅与Q有关, 不变.

$\oint\vec{D}\cdot d\vec{S}=Q_0$. $\vec{D}\cdot\Delta S=\sigma_e S$. $\vec{D}=\sigma_e=\frac{Q}{S}$

$\vec{D}=\varepsilon_0\vec{E}+\vec{P}$ $P=\frac{\varepsilon_0(\varepsilon_r-1)D}{\varepsilon_0\varepsilon_r}=(1-\frac{1}{\varepsilon_r})D$

13. 取r的方向为方向. $R_1<r<R_2$

$D\cdot2\pi r l=\lambda_0 l$, $D=\frac{\lambda_0}{2\pi r}\hat{e}_r$

$\vec{E}=\frac{D}{\varepsilon}=\frac{\lambda_0}{2\pi\varepsilon r}\hat{e}_r$

$\vec{P}=\vec{D}-\varepsilon_0\vec{E}=\frac{\lambda_0}{2\pi r}\hat{e}_r(1-\frac{\varepsilon_0}{\varepsilon})$

$\rho'=\nabla\cdot\vec{P}=0$, $\sigma'=\frac{\lambda_0}{2\pi r}(\frac{\varepsilon_0}{\varepsilon}-1)$

$\Rightarrow\sigma_1'=\frac{\lambda_0}{2\pi R_1}(\frac{\varepsilon_0}{\varepsilon}-1)$

$\sigma_2'=\frac{\lambda_0}{2\pi R_2}(\frac{\varepsilon_0}{\varepsilon}-1)$

$U=\int_{R_1}^{R_2}\frac{\lambda_0}{2\pi\varepsilon r}\hat{e}_r\cdot d\vec{r}=\frac{\lambda_0}{2\pi\varepsilon}\ln\frac{R_2}{R_1}$

$C=\frac{Q}{U}=\frac{\lambda_0 l}{\frac{\lambda_0}{2\pi\varepsilon}\ln\frac{R_2}{R_1}}=\frac{2\pi\varepsilon l}{\ln\frac{R_2}{R_1}}$

14. $\vec{E}\cdot4\pi r^2=\frac{Q}{\varepsilon_0}$, $E=\frac{Q(1+kr)}{4\pi r^2\varepsilon_0}$

$C=\frac{Q}{V}=\frac{Q}{Ed}=\frac{Q}{\int\frac{Q}{4\pi r^2}dr}=\frac{4\pi}{\ln\frac{b}{a}}$

$V=\int_a^b\vec{E}\cdot d\vec{r}=\frac{Q}{4\pi\varepsilon_0}(\frac{1}{a}-\frac{1}{b}+k\ln\frac{b}{a})$. $C=\frac{Q}{V}$

$\vec{P}=\vec{D}-\varepsilon_0\vec{E}=\frac{Q}{4\pi r^2}\vec{e}_r-\frac{Q}{4\pi r^2}(1+kr)\vec{e}_r$

$=-\frac{Q}{4\pi r^2}kr\vec{e}_r=-\frac{Q}{4\pi r}k\vec{e}_r$

$\rho'=-\nabla\cdot\vec{P}=\frac{kQ}{4\pi r^2}$, $\sigma_a'=P(a)(\vec{e}_r)$ $\sigma_b'=P(b)\vec{e}_r$

15. $\vec{D}\times4\pi r^2=\frac{\frac{4}{3}\pi(r^3-R^3)}{\frac{4}{3}\pi(8R^3-R^3)}q_0=\frac{r^3-R^3}{7R^3}q_0$

$\vec{D}_r=\frac{r^3-R^3}{7R^3}\times\frac{1}{4\pi r^2}=\varepsilon_r\varepsilon_0\vec{E}$

$\vec{E}_r=\frac{(r^3-R^3)q_0}{56\pi R^3r^2\varepsilon_0}\vec{e}_r$

$\Delta U=\int_R^{2R}\vec{E}_r\,d\vec{r}=\int_R^{2R}\frac{q_0}{56\pi R^3\varepsilon_0}(r-\frac{R^3}{r^2})\cdot d\vec{r}$

$=\int\frac{q_0}{56\pi\varepsilon_0R^3}(r^2-\frac{R^3}{r})dr=\frac{q_0}{56\pi\varepsilon_0R^3}(\frac{1}{2}(4R^2-R^2)-R^3\ln\frac{2R}{R})$

$=\frac{q_0}{56\pi\varepsilon_0R^3}(R^2-R^3\ln2)$

自由电荷. $\frac{q_0}{\frac{4}{3}\pi(8R^3-R^3)}$

金属内感应电荷. q.

$\therefore D\cdot4\pi r^2=\varepsilon_r\varepsilon_0E_1\cdot4\pi r^2=q+\frac{r^3-R^3}{7R^3}q_0$.

$E_1=\frac{1}{8\pi\varepsilon_0}(\frac{q}{r^2}+\frac{q_0r}{7R^3}-\frac{q_0}{7r^2})$.

介质外 $E_2=\frac{q+q_0}{4\pi\varepsilon_0r^2}$.

$\int_R^{2R}E_1dr=\int_{2R}^{\infty}E_2dr$.

$\frac{1}{8\pi\varepsilon_0}\int_R^{2R}(\frac{q}{r^2}+\frac{q_0r}{7R^3}-\frac{q_0}{7r^2})dr=\frac{q+q_0}{4\pi\varepsilon_0}\int_{2R}^{\infty}\frac{dr}{r^2}$. $q=-\frac{10}{21}q_0$

$U=\int_{2R}^{\infty}E_2dr=$ ...

16. $\vec{D}=\varepsilon_0\vec{E}+\vec{P}=\varepsilon_r\varepsilon_0\vec{E}$.

$C=\frac{Q}{V}=\frac{\sigma S}{E_0d}=\frac{\sigma S}{\frac{\sigma d}{\varepsilon_0}}=\frac{\varepsilon_0S}{d}$.

$\vec{D}\cdot S=+\sigma S\to$ 极板内.

介质外 $\varepsilon_r=1$. $P=0$. $E=\frac{D}{\varepsilon_0}=100V/m$.

介质内 $\varepsilon_r=2$. $P=(1-\frac{1}{\varepsilon_r})D=4.43\times10^{-10}C/m^2$. $E=\frac{D}{\varepsilon_0\varepsilon_r}$.

$U_1=\int_0^x\vec{E}\cdot d\vec{l}=100x$

$U_2=U_1+\int_1^x E_2\cdot d\vec{l}=100+50(x-1)$

$U_3=U_2+\int_2^x E_3\,d\vec{l}$

17. $D\times4\pi r^2=Q$. $D=\frac{Q}{4\pi r^2}$ 只由自由电荷决定.

$R_1<r<r_0$, $E_1=\frac{Q}{4\pi\varepsilon_1\varepsilon_0r^2}$

$r_0<r<R_2$, $E_2=\frac{Q}{4\pi\varepsilon_2\varepsilon_0r^2}$.

$C=\frac{Q}{U}=\frac{Q}{\int_r^{R_1}\frac{Q}{4\pi\varepsilon_1r^2}dr+\int_{R_2}^{r}\frac{Q}{4\pi\varepsilon_2r^2}dr}=\frac{1}{\frac{1}{4\pi\varepsilon_1}(\frac{1}{R_1}-\frac{1}{r})+\frac{1}{4\pi\varepsilon_2}(\frac{1}{r}-\frac{1}{R_2})}$

$=\frac{4\pi}{\frac{1}{\varepsilon_1}(\frac{1}{R_1}-\frac{1}{r})+\frac{1}{\varepsilon_2}(\frac{1}{r}-\frac{1}{R_2})}$ $P_1=\frac{Q(\varepsilon_1-\varepsilon_0)}{4\pi\varepsilon_1R^2}$

$\sigma_1'=(-\vec{P}_1)=-(D-\varepsilon_1E_1)=\frac{Q}{4\pi r^2}-\frac{Q}{4\pi r^2}=0$

$\sigma_{12}'=\vec{P}_1(-\vec{e}_R)=\frac{Q(\varepsilon_1-\varepsilon_0)}{4\pi\varepsilon_1R_1^2}$ $\sigma_2'=\vec{P}_2(\vec{e}_R)=-\frac{Q(\varepsilon_2-\varepsilon_0)}{4\pi\varepsilon_2R_2^2}$

18. $E_1\sin\theta_1=E_2\sin\theta_2$.

$D_1=\varepsilon_1E_1$ $D_2=\varepsilon_2E_2$

$P_1=(\varepsilon_1-\varepsilon_0)E_1$ $P_2=(\varepsilon_2-\varepsilon_0)E_2$

$(-\vec{P}_1)(\vec{P}_2)=p_2-p_1=\varepsilon_2E_2-\varepsilon_1E_1=\sigma'=\frac{\sigma}{\varepsilon_1E_1\cos\theta_1}$

$\varepsilon_1E_1\cos\theta_1-\varepsilon_2E_2\cos\theta_2=\sigma$

$E_1\sin\theta_1=E_2\cos\theta_2$

19. $\oint_S\vec{D}\cdot4\pi r^2=q$

$=D_1 2\pi r^2+D_2 2\pi r^2$

$=\varepsilon_1E\times2\pi r^2+\varepsilon_2E\times2\pi r^2=q$.

$E=\frac{q}{2\pi r^2}(\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_2})$, $D_1=\frac{\varepsilon_1q}{2\pi r^2}(\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_2})$, $D_2=\frac{\varepsilon_2q}{2\pi r^2}(\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_2})$

$\sigma_1=D_1|_R=\frac{\varepsilon_2q}{2\pi R^2}(\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_2})$, $e_1=$ —

20. (1) $\vec{E}=\frac{U}{d}$. $Q=CU$. $E=\frac{V}{d}$不变. $E_{\text{介}}=\frac{\sigma S}{\varepsilon_0}$

$\vec{E}_B=\frac{E_0}{\varepsilon}$. $\vec{E}_A=E_0$ $E_B<E_B$.

U不变.

(2) Q不变. $Q=CU$. $C=\frac{Q}{V}=\frac{\sigma S}{Ed}=\frac{\sigma S}{\frac{\sigma}{\varepsilon}d}=\frac{\varepsilon S}{d}$

21. $-q$受力: 受q, q内圆, q内圆.

$\vec{r}_3$ $(x,0,-c)$ ... 0.

D分布. $D=\frac{q}{4\pi r^2}$ 上 $E=\frac{q}{4\pi r^2\varepsilon_1}$, 下 $E=\frac{q}{4\pi r^2\varepsilon_2}$

$P_{上}=D-\varepsilon_0E_{上}=\frac{q}{4\pi r_1^2}-\varepsilon_0\frac{q}{4\pi r_1^2\varepsilon_2}=\frac{q}{4\pi r_1^2}(1-\frac{\varepsilon_0}{\varepsilon_2})$

$P_{下}=D-\varepsilon_0E_{下}=\frac{q}{4\pi r_1^2}-\varepsilon_0\frac{q}{4\pi r_1^2\varepsilon_1}=\frac{q}{4\pi r_1^2}(1-\frac{\varepsilon_0}{\varepsilon_1})$

$\rho=\nabla\cdot P=-\frac{1}{r}\frac{q}{4\pi}(1-\frac{\varepsilon_0}{\varepsilon})$

22. $\vec{E}_1=\frac{q}{4\pi\varepsilon_0r_1^3}\vec{r}_1=\frac{q}{4\pi\varepsilon_0(x-a,y-b)^3}(x-a,y-b)$

$\vec{E}_2=\frac{-q}{4\pi\varepsilon_0}\frac{(x+a,y-b)}{(x+a,y-b)^3}$

$\vec{E}_3=\frac{q}{4\pi\varepsilon_0}\frac{(x+a,y+b)}{(x+a,y+b)^3}$

$\vec{E}_4=\frac{-q}{4\pi\varepsilon_0}\frac{(x-a,y+b)}{(x-a,y+b)^3}$

$\vec{E}=\frac{q}{4\pi\varepsilon_0}\left(\frac{(x-a,y-b)}{r_1^3}-\frac{(x+a,y-b)}{r_2^3}+\frac{(x+a,y+b)}{r_3^3}-\frac{(x-a,y+b)}{r_4^3}\right)$

$E_y=\frac{q}{4\pi\varepsilon_0}\left(\frac{y-b}{r_1^3}-\frac{y-b}{r_2^3}+\frac{y+b}{r_3^3}-\frac{y+b}{r_4^3}\right)=0$,

$\frac{y-b}{r_1^3}+\frac{y+b}{r_3^3}=\frac{y-b}{r_2^3}+\frac{y+b}{r_4^3}$ $x=0$时, $r_1=r_2$, $r_3=r_4$

∴就上方两个导体平板.

另一个: $\frac{\sigma S}{\varepsilon_0}=E_yS$. $\sigma=\frac{q}{4\pi\varepsilon_0}\left(\frac{y-b}{r_1^3}-\frac{y-b}{r_2^3}+\frac{y+b}{r_3^3}-\frac{y+b}{r_4^3}\right)\varepsilon_0$

$=\frac{qb}{2\pi}\left(\frac{1}{[(x+a)^2+b^2+z^2]^{3/2}}-\frac{1}{[(x-a)^2+b^2+z^2]^{3/2}}\right)$

23. $\frac{q}{4\pi\varepsilon_0(R-d)}=\frac{q'}{4\pi\varepsilon_0(d'-R)}$

$\frac{q}{4\pi\varepsilon_0(R+d)}=\frac{q'}{4\pi\varepsilon_0(d'+R)}$

$qd'-qR=q'R-q'd$ $q'=\frac{R}{d}q$

$qd'+qR=q'R+q'd$ $d'=(\frac{R}{d})^2d=\frac{R^2}{d}$.

$E_1=\frac{q}{4\pi\varepsilon_0r_1^2}-\frac{q'}{4\pi\varepsilon_0r_2^2}$

$=\frac{1}{4\pi\varepsilon_0}(\frac{q}{r_1^2}-\frac{q'}{r_2^2})=\frac{1}{4\pi\varepsilon_0}($

$U_1=\frac{1}{4\pi\varepsilon_0}(\frac{q}{r_1}-\frac{q'}{r_2})$

带电量Q. $q'=\frac{R}{d}q$, $d'=\frac{R^2}{d}$ E_1. U_1.

$E_2=0$. $U_2=\frac{Q}{4\pi\varepsilon_0R}$

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1. (1) $ES=\frac{\sigma S}{\varepsilon_0}$, $E=\frac{\sigma}{\varepsilon_0}$ (σ有正负)

$E_1+E_2=0=\frac{(\sigma_2+\sigma_3)S}{\varepsilon_0}$, $\sigma_2=-\sigma_3$.

取向右为正. $\frac{\sigma_1}{\varepsilon_0}-\frac{\sigma_2}{\varepsilon_0}-\frac{\sigma_3}{\varepsilon_0}-\frac{\sigma_4}{\varepsilon_0}=0$.

$\frac{\sigma_1}{\varepsilon_0}+\frac{\sigma_2}{\varepsilon_0}+\frac{\sigma_3}{\varepsilon_0}-\frac{\sigma_4}{\varepsilon_0}=0$

$\therefore\sigma_1=\sigma_4$.

(2) $\sigma_1S+\sigma_2S=Q_1$, $\sigma_3S+\sigma_4S=Q_2$. $\Rightarrow$ $\sigma_1S+\sigma_2S=Q$, $-\sigma_2S+\sigma_1S=Q_2$

$\sigma_1=\frac{Q_1+Q_2}{2S}=\sigma_4$, $\sigma_2=\frac{Q_1-Q_2}{2S}=\sigma_3$.

2. $0<r<R_1$, $E_1=0$.

$R_1<r<R_2$, $E_2=\frac{q}{4\pi r^2\varepsilon_0}$.

$R_2<r<R_3$, $E_3=0$.

$r>R_3$, $E_4=\frac{q+Q}{4\pi r^2\varepsilon_0}$

内球电势为0. 相无穷远电势为0

$\int_{R_3}^{\infty}\vec{E}_4\cdot d\vec{l}+\int_{R_2}^{R_3}\vec{E}_3\cdot d\vec{l}+\int_{R_1}^{R_2}\vec{E}_2\cdot d\vec{l}=0$

$\frac{q+Q}{4\pi\varepsilon_0}\int_{R_3}^{\infty}\frac{1}{r^2}dr+\frac{q}{4\pi\varepsilon_0}\int_{R_1}^{R_2}\frac{1}{r^2}d\vec{l}=0$

$-(q+Q)\times\frac{1}{R_3}=q\times(\frac{1}{R_2}-\frac{1}{R_1})$

$q(\frac{1}{R_1}-\frac{1}{R_2}+\frac{1}{R_3})=-Q\times\frac{1}{R_3}$.

$q=\frac{-\frac{1}{R_3}Q}{\frac{1}{R_1}-\frac{1}{R_2}+\frac{1}{R_3}}=\frac{-R_1R_2Q}{R_2R_3-R_1R_3+R_1R_2}$.

球壳电势 $\frac{q+Q}{4\pi\varepsilon_0}\int_{R_3}^{\infty}\frac{1}{r^2}dr$

$=-\frac{q+Q}{4\pi\varepsilon_0}(0-\frac{1}{R_3})$

$=\frac{1}{4\pi\varepsilon_0}\frac{R_2R_3Q+R_1R_2Q-R_1R_3Q-R_1R_2Q}{R_1R_3-R_2R_3-R_1R_2}\times\frac{1}{R_3}$

$=\frac{1}{4\pi\varepsilon_0}\frac{(R_2-R_1)Q}{R_2R_3-R_1R_3-R_1R_2}$

3. $\Delta T=\int2\alpha\,dl\sin\frac{\theta}{2}$

$=2\alpha\,2\pi r\frac{\theta}{2}\sin\frac{\theta}{2}$

$=\alpha\pi r$.

原: $\frac{\Delta F}{\Delta S}=p=\frac{\Delta T}{2\pi(r\frac{\theta}{2})^2}=\frac{\alpha 2r}{2r^2\frac{\theta^2}{4}}=\frac{4\alpha}{r}$.

后: $F_c=(E_3-E_{d_q})dq=(\frac{\sigma}{\varepsilon_0}-\frac{\sigma}{2\varepsilon_0})dq$

$=\frac{1}{2\varepsilon_0}\times\frac{Q}{4\pi R^2}dq$

$pV=nRT$. $\frac{p_1}{p_2}=\frac{V_2}{V_1}$.

$p_2=\frac{V_1}{V_2}p_1=\frac{r^3}{R^3}p$.

$(\frac{r^3}{R^3}-1)p\Delta S=\frac{1}{2\varepsilon_0}\frac{Q}{4\pi R^2}\frac{Q}{4\pi R^2}\Delta S$

$(R^3-r^3)p=\frac{1}{2\varepsilon_0}\times\frac{Q^2}{16\pi^2R}$

$p+\frac{4\alpha}{r}$

$(\frac{r^3}{R^3}-1)(p+\frac{4\alpha}{r})=-\frac{Q^2}{32\pi^2\varepsilon_0R^4}$.

5. 所测在A上: $\vec{F}=\frac{1}{4\pi\varepsilon_0}\frac{(q_1+q_2)q}{r^3}\vec{r}$ $(-\vec{F}_A)$

q_1, q_2不受q的作用.

q_1, q_2受A的感应电荷.

中心一致q_0为0

6. (1) $C=\frac{Q}{V}=\frac{Q}{Ed}=\frac{\sigma S}{\frac{\sigma}{\varepsilon_0}d}=\frac{\varepsilon_0S}{d}$

$C_1=C_2=\frac{8.85\times10^{-12}\times6\times10^{-4}m^2}{0.1\times10^{-3}m}$

$=8.85\times6\times10^{-16}F=53.1\times10^{-10}F$.

$C=C_1+C_2=106.2\times10^{-10}F$.

(2) $Q=CV=106.2\times10^{-16}\times220$

拆除: $V_1=V_2$, $\frac{Q_1}{C_1}=\frac{Q_2}{C_2}=\frac{Q}{C_1+C_2}$

$Q=Q_1+Q_2=(C_1+C_2)V$

$\sigma_1=\frac{Q_1}{S}=\frac{53.1\times10^{-10}\times220}{6\times10^{-4}}=$ ... 两边σ_1, σ_2. 中间$(\sigma_1+\sigma_2)$

8. $U_4+U_5=U$ $C_4U_4=C_3U_3=C_5U_5$

$U_1+U_2=U$. $C_1U_1=C_3U_3+C_4U_4$

$U_1+U_3=U_4$

$U_3+U_5=U_2$

$U_4+U_5=600$ $U_4+U_3=2U_5$ $600+U_5=3U_5$

$U_1+U_2=600$ $U_2+U_3=2U_1$ $U_4+U_5+U_3=3U_5$

$U_2+U_3=U_4$ $600+U_3=3U_1$

$U_1=U_3$. $U_2=U_4$

$U_2+U_1=2U_4-2U_3$ $3U_3=U_2-U_4$

$3U_3+U_1=600$, $600+U_1=3U_1=1800-9U_3$

$12U_3=1200$. $U_1=120$. $U_2=480=U_4$, $U_1=240=U_3$

9. 并联.

$E\cdot2\pi rl=\frac{Q}{\varepsilon_0}$, $E=\frac{Q}{2\pi rl\varepsilon_0}$

$V=\int_a^b\vec{E}\cdot d\vec{l}=\frac{Q}{2\pi l\varepsilon_0}\int_a^b\frac{1}{r}dr=\frac{Q}{2\pi l\varepsilon_0}\ln\frac{b}{a}$

$C_1=\frac{Q}{V}=\frac{2\pi l\varepsilon_0}{\ln\frac{b}{a}}$

$C=C_1+C_2=2\pi l\varepsilon_0(\frac{1}{\ln(\frac{b}{c})}+\frac{1}{\ln(\frac{c}{a})})$

12. $C=\frac{Q}{V}=\frac{\sigma S}{Ed}=\frac{\sigma S}{\frac{\sigma}{\varepsilon_0}d}=\frac{\varepsilon_0S}{d}$ 不变.

$\varepsilon_1=5\varepsilon_0$, $\varepsilon_2=2\varepsilon_0$ $D\cdot S=\sigma S$, $D=\sigma$.

$D=P+\varepsilon E$.

接上撤电源, Q不变. $Q=CU=\frac{\varepsilon_0S}{d}V=4\varepsilon_0\times10^6$.

$D=\sigma=\frac{Q}{S}=2\varepsilon_0\times10^6C\cdot m^{-2}$.

$D=\varepsilon E=\varepsilon_0E+P=\varepsilon_r\varepsilon_0E$.

$P=\varepsilon_0E(\varepsilon_r-1)=\frac{\varepsilon_0D(\varepsilon_r-1)}{\varepsilon_0\varepsilon_r}\Rightarrow P_1$. P_2

$E_1=\frac{D}{\varepsilon_1}$. $E_2=\frac{D}{\varepsilon_2}$ $V=E_1d_1+E_2d_2=$ ___

13. (1) $D\times2\pi r\times l=\lambda_0l$, $\vec{D}=\frac{\lambda_0}{2\pi r}\hat{r}$.

$\vec{E}=\frac{\vec{D}}{\varepsilon}\hat{r}=\frac{\lambda_0}{2\pi r\varepsilon}\hat{r}$.

$\vec{P}=\vec{D}-\varepsilon_0\vec{E}=\vec{D}(1-\frac{\varepsilon_0}{\varepsilon})=\frac{\lambda_0(\varepsilon-\varepsilon_0)}{2\pi r\varepsilon}\hat{r}$.

$\rho'=-\nabla\cdot\vec{P}=\frac{\lambda_0(\varepsilon-\varepsilon_0)}{2\pi r\varepsilon}$

$-\frac{1}{r}\frac{\partial}{\partial r}[r\cdot\frac{\lambda_0(\varepsilon-\varepsilon_0)}{2\pi\varepsilon r}]=0$

$\vec{D}\cdot\vec{S}=DS=\sigma S$.

$\sigma_1=D_1=\frac{\lambda_0}{2\pi R_1}$, $\sigma_2=D_2=\frac{\lambda_0}{2\pi R_2}$.

(2) $V=\int_{R_1}^{R_2}\vec{E}\cdot d\vec{r}=\frac{\lambda_0}{2\pi\varepsilon}\ln\frac{R_2}{R_1}$

(3) $C=\frac{\lambda_0l}{V}=\frac{\lambda_0l}{\frac{\lambda_0}{2\pi\varepsilon}\ln\frac{R_2}{R_1}}=\frac{2\pi\varepsilon l}{\ln\frac{R_2}{R_1}}$

14. $\varepsilon=\frac{\varepsilon_0}{1+kr}$.

$D\cdot4\pi r^2=Q$. $D=\frac{Q}{4\pi r^2}$

$E=\frac{D}{\varepsilon}=\frac{Q}{4\pi r^2\varepsilon}=\frac{Q}{4\pi\varepsilon_0}\frac{1+kr}{r^2}$

$V=\int_a^b\frac{Q}{4\pi\varepsilon_0}\frac{1+kr}{r^2}dr=\frac{Q}{4\pi\varepsilon_0}\int_a^b\frac{1}{r^2}+\frac{k}{r}dr$

$=\frac{Q}{4\pi\varepsilon_0}(\frac{1}{a}-\frac{1}{b}+k\ln\frac{b}{a})$ ✓

$C=\frac{Q}{V}=\frac{4\pi\varepsilon_0}{\frac{1}{a}-\frac{1}{b}+k\ln\frac{b}{a}}$ ✓

$\vec{P}=\vec{D}-\varepsilon_0\vec{E}=D(1-\frac{\varepsilon_0}{\varepsilon})=\frac{Q}{4\pi r^2}(1-\frac{\varepsilon_0}{\frac{\varepsilon_0}{1+kr}})=-\frac{kQ}{4\pi r}\hat{r}$

$\rho'=-\nabla\cdot\vec{P}=\frac{kQ}{4\pi r^2}$ $\frac{\partial}{\partial r}$.

$\sigma_a=D_a=\frac{Q}{4\pi a^2}$, $\sigma_b=D_b=\frac{Q}{4\pi b^2}$.

$\sigma_a'=\vec{P}(a)\cdot(-\vec{e}_r)=\frac{kQ}{4\pi a}$.

15. $D\times4\pi r^2=Q$.

$D=\begin{cases}\frac{q_0}{4\pi r^2}, & r\geq2R\\ \frac{(\frac{4}{3}\pi r^3-\frac{4}{3}\pi R^3)\frac{q_0}{\frac{4}{3}\pi 8R^3-\frac{4}{3}\pi R^3}}{4\pi r^2}=\frac{(r^3-R^3)q_0}{4\pi r^2\times7R^3}, & R<r<2R\\ 0, & r<R.\ ?\end{cases}$

无自由电荷. 感应电荷q. q会影响E.

$\vec{E}=\frac{q_0(r^3-R^3)}{4\pi r^2\times7R^3\times2\varepsilon_0}$. $U=\int\vec{E}\cdot d\vec{r}$ $4\pi r^2\cdot r^2=3R^3$

$U=\frac{q_0}{4\pi\times7R^3\times2\varepsilon_0}\int_R^{2R}\frac{r^3-R^3}{r^2}dr$

$=\frac{1}{8\pi\varepsilon_0}\times\frac{q_0}{7R^3}(\frac{1}{2}r^2\Big|_R^{2R}+R^3(\frac{1}{2R}-\frac{1}{R}))$

$=\frac{1}{8\pi\varepsilon_0}\times\frac{q_0}{7R^3}(\frac{3}{2}R^2-\frac{1}{2}R^2)$

16. $DS=+\sigma S$, $D=-\sigma\vec{x}$.

$0\leq x\leq1$, $V=\int_0^x\frac{\sigma}{\varepsilon_0}dl=\frac{\sigma x}{\varepsilon_0}$

$1\leq x\leq2$, $V=\int_0^1\frac{\sigma}{\varepsilon_0}dl+\int_1^x\frac{\sigma}{2\varepsilon_0}dl$

$=\frac{\sigma}{\varepsilon_0}+\frac{\sigma(x-1)}{2\varepsilon_0}$

$E=\frac{D}{\varepsilon_0}$ $E=\frac{D}{2\varepsilon_0}$ $E=\frac{D}{\varepsilon_0}$ $2\leq x\leq3$, $V=\int_0^1\frac{\sigma}{\varepsilon_0}dl+\int_1^2\frac{\sigma}{2\varepsilon_0}dl+\int_2^x\frac{\sigma}{\varepsilon_0}dl$.

$P=0$ $P=\frac{D}{2}$ $P=0$ $=\frac{\sigma}{\varepsilon_0}+\frac{\sigma}{2\varepsilon_0}+\frac{\sigma}{\varepsilon_0}(x-2)$

17. 内球带Q. $\frac{Q}{4\pi a^2}-\frac{\varepsilon_0}{\varepsilon_1}(\frac{Q}{4\pi a^2})$

$D=\begin{cases}\frac{\frac{a^3}{R_1^3}Q}{4\pi a^2}, & 0<a\leq R_1\\ \frac{Q}{4\pi a^2}, & R_1<a\leq R_2\end{cases}$

$E=\begin{cases}\frac{aQ}{4\pi R_1^3\varepsilon_1}, & 0<a\leq R_1\\ \frac{Q}{4\pi a^2\varepsilon_1}, & R_1<a\leq r\\ \frac{Q}{4\pi a^2\varepsilon_2}, & r<a\leq R_2\end{cases}$

$V_{R_1R_2}=\frac{Q}{4\pi}(\frac{1}{\varepsilon_1}(\frac{1}{R_1}-\frac{1}{r})+\frac{1}{\varepsilon_2}(\frac{1}{r}-\frac{1}{R_2}))$

$C=\frac{Q}{V}=\frac{4\pi}{\frac{1}{\varepsilon_1}(\frac{1}{R_1}-\frac{1}{r})+\frac{1}{\varepsilon_2}(\frac{1}{r}-\frac{1}{R_2})}$

$P=D-\varepsilon_0E$ $\vec{P}_1=\frac{-Q(\varepsilon_1-\varepsilon_0)}{4\pi\varepsilon_1R^2}\vec{e}_R$, $\vec{P}_2=\frac{-Q(\varepsilon_2-\varepsilon_0)}{4\pi\varepsilon_2R^2}\vec{e}_R$.

$\sigma_r=(-\vec{P}_2-(-\vec{P}_1))(\vec{n}_1-\vec{n}_2)$

$=(\vec{P}_1-\vec{P}_2)\vec{n}_{12}=\frac{-Q}{4\pi r^2}(\frac{\varepsilon_1-\varepsilon_0}{\varepsilon_1}-\frac{\varepsilon_2-\varepsilon_0}{\varepsilon_2})=\frac{-Q}{4\pi r^2}(\frac{\varepsilon_0}{\varepsilon_2}-\frac{\varepsilon_0}{\varepsilon_1})$ ✓

$\sigma_1=-\vec{P}_1\vec{n}_1=\frac{Q(\varepsilon_1-\varepsilon_0)}{4\pi\varepsilon_1R_1^2}$, $\sigma_2=-\vec{P}_2\cdot\vec{n}_2=\frac{Q(\varepsilon_2-\varepsilon_0)}{4\pi\varepsilon_2R_2}$.