课后题

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1. $\vec{F}=\frac{q_1q_2}{4\pi\varepsilon_0 r^2}\frac{\vec{r}}{r}$, $F=\frac{q_1(q-q_1)}{4\pi\varepsilon_0 r^2}$, $\frac{dF}{dq_1}=0\Rightarrow q_1=\frac{q}{2}$.

$\vec{F}_{23}=G\frac{Mm}{r^2}=6.67\times10^{-11}\times\frac{5.98\times10^{24}\times7.34\times10^{22}}{r^2}$

$=3.99\times10^{20}\cdot\frac{60^2}{r^2}=6.67\times5.98\times7.34\times10^{-11}$

$\Omega_{sun}=\frac{2\pi}{365\times24\times3600}$, $\omega_{23}=\Omega_{sun}$, $\frac{G(m_2+m_3)}{r^3}=\omega_{23}^2$

$r=\sqrt[3]{\frac{G(m_2+m_3)}{\omega_{23}^2}}=\sqrt[3]{\frac{6.67\times10^{-11}\times(5.98+7.34)\times10^{24}}{(\frac{2\pi}{365\times24\times3600})^2}}=\frac{\sqrt[3]{6.67\times13.32}}{2\pi}\times365\times24\times3600\times10^{8}$

$\approx5.98\times10^{10}=0.4\times10^{11}$

2. $\vec{F}_{12}=\frac{1}{4\pi\varepsilon_0}\frac{|Q_1q|}{r^2}\vec{r}$, $\vec{F}=m\frac{d^2\vec{r}}{dt^2}$, $\frac{d^2\vec{r}}{dt^2}=\frac{q|Q|}{|Q|4\pi\varepsilon_0 r^2}\vec{r}$.

3. $\vec{F}_{21}=k\frac{q_2q_1}{a^2}\vec{i}$, $\vec{F}_{31}=k\frac{q_3q_1}{a^2}\vec{j}$, $\vec{F}_{21}+\vec{F}_{31}=\vec{F}_{41}$

$\vec{F}_{41}=k\frac{q_4q_1}{(\sqrt{2}a)^2}\vec{r}_0$, $\vec{r}_{41}=\frac{q_4q_1}{4\pi\varepsilon_0 a^2}\frac{\vec{a}}{\sqrt{2}a}=\frac{\vec{a}}{2\sqrt{2}a}$, $\vec{r}_0=\frac{\vec{a}}{\sqrt{2}a}$.

$=-\frac{1}{8\sqrt{2}}\vec{a}$.

$V=\int\frac{dq}{4\pi\varepsilon_0 r}=\int\frac{\lambda dl}{4\pi\varepsilon_0 a}=\frac{\lambda\cdot2\pi a}{4\pi\varepsilon_0 a}=\frac{\lambda}{2\varepsilon_0}$.

4. $\vec{F}_{21}=\vec{F}_{31}=k\frac{q_2q_1}{(\sqrt{2}r)^2}$, $\theta=\frac{\pi}{4}$, $\vec{F}=2\times k\frac{q_2q_1}{(\sqrt{2}r)^2}\cdot\frac{r}{\sqrt{2}r}\Rightarrow F(r)=2\cdot\frac{kq_2q_1}{(\sqrt{2}r)^3}r$.

$F'(r)=2kq_2q_1\frac{(\sqrt{2}r)^3-r\cdot3(\sqrt{2}r)^2\sqrt{2}r}{(\sqrt{2}r)^6}$

$=2kq_2q_1\frac{(\sqrt{2}r)^2-3r^2\sqrt{2}r}{(\sqrt{2}r)^4}=0$,

$(\sqrt{2}r)^2=2(r^2+h^2)^{\frac{3}{2}}$, $2r^2=2r^3$, $r=\sqrt{2}h$

5. $\vec{E}=k\frac{q}{(\sqrt{2}r)^2+r^2}\cdot\frac{1}{\sqrt{2}}\vec{i}$, $\vec{E}_0=k\frac{q}{(\sqrt{2}r)^2+r^2}\cdot\frac{r}{\sqrt{2}r}\vec{j}$, $\vec{E}=\vec{E}_0$

$\vec{E}_1+\vec{E}_2=2k\phi\left(\frac{1}{(\frac{1}{2}-\frac{1}{2n+1})^2}-\frac{1}{(\frac{1}{2}+\frac{1}{2n+1})^2}\right)=\left(\frac{1}{n^2}-\frac{1}{n+1}\right)^2$

$n\gg1$, $\vec{E}_1+\vec{E}_2=2k\phi\left(\frac{1}{n^2}-\frac{1}{n^3}\right)=\frac{4k\phi}{n^3}\vec{i}$

6. 对称性: $dq=\lambda Rd\theta\cdot dl$, $dl=dR\theta=Rd\theta$

$d\vec{E}=k\frac{dq}{R^2}d\theta$ $dq=\lambda_0\theta\,Rd\theta$

$\therefore E=\int_0^{\frac{\pi}{2}}k\frac{\lambda_0}{R^2}\theta\,Rd\theta$ $\frac{k=\frac{1}{4\pi\varepsilon_0}}{}$

$=\frac{\lambda_0\theta}{4\pi\varepsilon_0}\left(\frac{\theta}{2}-\frac{\sin2\theta}{4}\right)\Big|_0^{\frac{\pi}{2}}=\frac{\lambda_0\pi}{16\varepsilon_0}$

7. 建系 $\vec{i}$向右 $\vec{j}$向下 $\vec{k}$向外

$E_1=\int_0^{\frac{\pi}{2}}\frac{k}{R^2}\cos\theta\,dq=\int_0^{\frac{\pi}{2}}\frac{k}{R^2}\cos\theta\,\lambda Rd\theta$

$=\frac{k\lambda}{R}\int_0^{\frac{\pi}{2}}\cos\theta\,d\theta=\frac{k\lambda\cdot1}{R}$.

$E_2=-2\pi k\frac{h}{2\pi R^2}\lambda\int_0^{\frac{\pi}{2}}\sin\theta\,d\theta$

$=-2\times k\frac{h}{R^2}\lambda\int_0^{\frac{\pi}{2}}\sin\theta\,d(\cos\theta)$

$=-2\times k\frac{h}{R^2}\lambda\cos\theta\Big|_0^{\frac{\pi}{2}}=-\frac{k\lambda h}{R^2}$

$\vec{E}=(\vec{B}\cdot\nabla)\vec{B}$

$\vec{E}=\vec{E}_1+\vec{E}_2=\frac{k\lambda}{R}\vec{i}-\frac{k\lambda h}{R^2}\vec{j}$

$\vec{E}=0$

$(\vec{B}\cdot\nabla)\vec{B}=(B_x\frac{\partial}{\partial x}+B_y\frac{\partial}{\partial y}+B_z\frac{\partial}{\partial z})\vec{B}$

$\vec{E}=\nabla(\vec{B}\cdot\vec{B})$

$\vec{E}=\nabla\frac{B^2}{2}$

$\vec{E}=\nabla\frac{|\vec{B}|^2}{2}$

$\vec{E}=\nabla\frac{|\vec{B}|^2}{2}$

8. $F_+=\frac{kQq}{(D-\frac{L}{2})^2}$, $F_-=\frac{kQq}{(D+\frac{L}{2})^2}$

$F=F_+-F_-=kQq\left(\frac{1}{(D-\frac{L}{2})^2}-\frac{1}{(D+\frac{L}{2})^2}\right)$, $M\gg L$

$=kQq\left(\frac{1}{D^2(1-\frac{L}{2D})^2}-\frac{1}{D^2(1+\frac{L}{2D})^2}\right)$

$\frac{1}{(1-x)^2}=1+2x+3x^2+\cdots$

$\frac{1}{(1+x)^2}=1-2x+3x^2-\cdots$

$\approx\frac{kQq}{D^2}\left(1+\frac{L}{D}-1+\frac{L}{D}\right)=\frac{kQqL}{D^3}$

$\vec{F}=\frac{kQq}{D^3}\vec{L}$, $\vec{M}=\vec{P}_m\times\vec{B}=\frac{kQqL}{D^3}\vec{j}\times\frac{k}{D^2}\vec{i}$

$\vec{L}=\frac{\vec{M}}{kQq}=\frac{\vec{P}_m\times\vec{B}}{kQq}$, $\Rightarrow\vec{F}=\frac{\vec{P}_m\times\vec{B}}{D^3}\cdot\frac{1}{kQq}=\frac{\vec{P}_m\times\vec{B}}{4\pi\varepsilon_0 D^3}$

$\vec{M}=\vec{L}\times\vec{L}=2\pi\cdot\frac{1}{4\pi\varepsilon_0}\cdot\frac{-q\vec{j}}{D^2}\times\frac{k}{D^2}\vec{i}=\frac{1}{2}\times\frac{-q\vec{j}}{4\pi\varepsilon_0 D^2}\times\vec{i}$

$=\frac{Dq\vec{k}}{4\pi\varepsilon_0(D^2+\frac{L^2}{4})^2}$

9. $R^2=(R\cos\theta)^2+r^2=R(R+d)\cos\theta$

$r^2=2R^2+d^2-2dR\cos\theta$

$dq=\sigma dS=\sigma\cdot2\pi(R-R\cos\theta)\cdot R\,d\theta$

$=2\pi\sigma R^2(1-\cos\theta)d\theta$

$\vec{E}=\int_0^{\frac{\pi}{2}}k\frac{dq}{r^2}=\int_0^{\frac{\pi}{2}}k\frac{2\pi\sigma R^2}{2R^2+d^2-2Rd\cos\theta}d\theta$

Gauss $4\pi kR^2\sigma=\oint\vec{E}\cdot d\vec{S}=\oint\vec{E}\cdot\vec{n}\,dS$

$P_1\sigma=P_2$ $4\pi kP_1^2\sigma=4\pi kP_2^2\sigma=\oint\vec{E}\cdot\vec{n}\,dS$

10. $d\tau=dx\,R\,d\varphi\,dR$, $dq=\rho d\tau$

$E=\frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r^2}$

$=\frac{\rho}{4\pi\varepsilon_0}\int_0^R\int_0^{2\pi}\int_{-\infty}^{+\infty}\frac{1}{(x+a)^2}\,dx\,R\,d\varphi\,dR$

$=\frac{R\rho}{4\pi\varepsilon_0}\int_0^R\int_0^{2\pi}\int_{-\infty}^{+\infty}\frac{1}{(x+a)^2+R^2}\,dx\,d\varphi\,dR$

$=\frac{R\rho}{2\varepsilon_0}\int_0^R\int_{-\infty}^{+\infty}\frac{1}{(x+a)^2+R^2}\,dx\,dR$

$=\frac{R\rho}{2\varepsilon_0}\int_0^R\int_{-\infty}^{+\infty}\frac{1}{(x+a)^2+R^2}\,d(x+a)\,dR$

若干薄板叠加: $\sigma=\rho dx$

$E=\int_0^R\frac{k\pi\rho dx}{r^2}=\frac{k\pi\sigma}{r^2}$, $r\gg R$

$E=\int_0^{\infty}\frac{k\pi\sigma dx}{r^2}=-\frac{k\pi\sigma}{r}\Big|_0^{\infty}=\frac{k\pi\sigma}{r}$

$E=\int_0^R\frac{k\pi\rho dx}{r^2}=-\int_0^R k\pi\rho\frac{d(a-x)}{(a-x)^2}$, $r=a-x$

$=k\pi\rho\frac{1}{a-x}\Big|_0^R=k\pi\rho\left(\frac{1}{a-R}-\frac{1}{a}\right)$

$=\frac{k\pi\rho R}{a(a-R)}=\frac{k\pi\sigma}{a}$, $r=a$

$E=\int_R^{\infty}\frac{k\pi\rho R}{r^2}dr=-k\pi\rho R\frac{1}{r}\Big|_R^{\infty}=k\pi\rho$, $r=r=R$

11. $d\tau=dz\,r\,d\varphi\,dr$, $dq=\rho d\tau$

$Q=\int_0^{\infty}\int_0^{2\pi}\int_0^{\infty}-\frac{q_0}{\pi a^4}e^{-\frac{2r}{a}}r\,d\varphi\,dr\,dz$

$=-\frac{2\pi q_0}{\pi^2}\int_0^{\infty}dz\times\int_0^{\infty}e^{-\frac{2r}{a}}r\,dr$

$=-\frac{2q_0}{\pi}\int_0^{\infty}e^{-\frac{2r}{a}}r\,dr$

$\int e^{kr}r\,dr=\frac{1}{k}\int r\,de^{kr}=\frac{1}{k}[e^{kr}r-\int e^{kr}dr]$

$=\frac{1}{k}r^2e^{kr}-\frac{2}{k}\int re^{kr}dr=\frac{1}{k}re^{kr}-\frac{2}{k^2}(re^{kr}-\int e^{kr}dr)$

$=\frac{1}{k}r^2e^{kr}-\frac{2}{k^2}re^{kr}+\frac{2}{k^3}e^{kr}$

$=-\frac{2q_0}{\pi}(-\frac{a}{2}r^2e^{-\frac{2r}{a}}\Big|_0^{\infty}-\frac{a}{2}r\cdot2e^{-\frac{2r}{a}}\Big|_0^{\infty}+\frac{a^2}{4}e^{-\frac{2r}{a}}\Big|_0^{\infty})$

$=-\frac{2q_0}{\pi}(-\frac{a}{2}\cdot0^2\cdot e^{-\frac{2\cdot0}{a}}-\frac{a}{2}\cdot0\cdot e^{-\frac{2\cdot0}{a}}+\frac{a^2}{4}e^{-\frac{2\cdot0}{a}}-\frac{a}{2}\cdot\infty^2\cdot e^{-\frac{2\cdot\infty}{a}}-\frac{a}{2}\cdot\infty\cdot e^{-\frac{2\cdot\infty}{a}}-\frac{a^2}{4}e^{-\frac{2\cdot\infty}{a}})$

$=\pi\cdot a\,q_0\cdot e^{-2}+\pi\cdot a\,q_0\cdot e^{-2}+\frac{1}{2}\pi\,a\,q_0\,e^{-2}-\frac{1}{2}\pi\,a\,q_0$

$=\frac{3}{2}\pi\,a\,q_0\,e^{-2}-\frac{1}{2}\pi\,a\,q_0$

$=\frac{1}{2}\pi\,a\,q_0(3e^{-2}-1)$

$\Phi=\oint\vec{E}\cdot d\vec{S}=\frac{Q}{\varepsilon_0}=\frac{1}{2}\pi a\,q_0(3e^{-2}-1)$

12. $\vec{r}_0=\vec{x}-\vec{y}$ $0<x<r_1$, $\vec{E}=\frac{\vec{r}_0}{S\varepsilon_0}$

$E_1=\frac{\pi x^2\sigma}{4\pi\varepsilon_0}=\frac{\rho r_1}{3\varepsilon_0}$

$E_2=\frac{\rho r_2}{3\varepsilon_0}$ $\vec{x}+\vec{r}_2=\vec{r}_1$.

$\vec{E}=\frac{\rho}{3\varepsilon_0}(\vec{r}_1-\vec{r}_2)$

13. $\lambda_0=\lambda_0x$

$E=\frac{\sigma}{2\varepsilon_0}=\frac{\lambda dx}{2\varepsilon_0 dx}=\frac{\lambda}{2\varepsilon_0}$

$E_{(0,0)}=\frac{\lambda_0(x=0)}{2\varepsilon_0}=\frac{\lambda_0\cdot0}{2\varepsilon_0}=\frac{\lambda_0a}{2\varepsilon_0}=\frac{\lambda_0x}{2\varepsilon_0}$

$E_{(a,0)}=\frac{\lambda_0a}{2\varepsilon_0}-\frac{\lambda_0\cdot0}{2\varepsilon_0}=\frac{\lambda_0(x=a)}{2\varepsilon_0}$

14. $\rho(x)=\frac{\rho_0}{a}x$

$E=\int\frac{dx}{4\pi\varepsilon_0(a+b-x)^2}\cdot\frac{\rho_0}{a}x$

$=\frac{\rho_0}{4\pi\varepsilon_0 a}\int_0^a\frac{x}{(a+b-x)^2}dx$

$=\frac{\rho_0}{4\pi\varepsilon_0 a}\int_0^a\frac{x}{(x-(a+b))^2}dx$

$=\frac{\rho_0}{4\pi\varepsilon_0 a}\int_0^a\frac{x-(a+b)+(a+b)}{(x-(a+b))^2}dx$

$=\frac{\rho_0}{4\pi\varepsilon_0 a}\int_0^a\left(\frac{1}{x-(a+b)}+\frac{a+b}{(x-(a+b))^2}\right)dx$

$=\frac{\rho_0}{4\pi\varepsilon_0 a}\left[\ln|x-(a+b)|-\frac{a+b}{x-(a+b)}\right]_0^a$

$=\frac{\rho_0}{4\pi\varepsilon_0 a}\left[\ln\frac{b}{a+b}-\frac{a+b}{a}+\frac{a+b}{b}\right]$

$=\frac{\rho_0}{4\pi\varepsilon_0 a}\left[\ln\frac{b}{a+b}+\frac{b}{a}+1\right]$

15. $\vec{E}=\frac{\sigma}{2\varepsilon_0}\vec{n}=\frac{\rho}{2\varepsilon_0}\vec{n}$, $\vec{E}=\lambda\rho\vec{n}=\frac{2k\pi\sigma}{\varepsilon_0}\vec{n}$

$\therefore\vec{E}=\frac{\rho}{2\varepsilon_0}\rho\vec{n}=\frac{k\pi\rho}{\varepsilon_0}\vec{n}$

16. $\oint\vec{E}\cdot d\vec{l}=0$ $\vec{E}=\vec{E}_{\text{外}}+\vec{E}_{\text{内}}$

$\oint\vec{E}_{\text{外}}\cdot d\vec{l}+\oint\vec{E}_{\text{内}}\cdot d\vec{l}=0$

$U_{\text{外}}-U_{\text{内}}=0$ $U_{\text{外}}=U_{\text{内}}$

17. $r_1^2=r^2+\frac{R^2}{4}-2r\cdot\frac{R}{2}\cos\theta$

$r_2^2=r^2+\frac{R^2}{4}+2r\cdot\frac{R}{2}\cos\theta$

$U_1=\frac{q}{4\pi\varepsilon_0 r_1}=\frac{q}{4\pi\varepsilon_0\sqrt{r^2+\frac{R^2}{4}-2r\cdot\frac{R}{2}\cos\theta}}$

$U_2=\frac{q}{4\pi\varepsilon_0\sqrt{r^2+\frac{R^2}{4}+2r\cdot\frac{R}{2}\cos\theta}}$

$r\ll R$ $U=\frac{2q}{4\pi\varepsilon_0 R}=\frac{q}{2\pi\varepsilon_0 R}$

$U_{\text{外}}=\frac{q}{4\pi\varepsilon_0}\left(\frac{1}{\sqrt{r^2-2r\cdot\frac{R}{2}\cos\theta}}-\frac{2}{R}r\right)=\frac{q}{2\pi\varepsilon_0}(3e^{-1}\cos\theta-1)$

$=\frac{q}{4\pi\varepsilon_0}\cdot1$

$\vec{E}=\nabla U=\cdots$

18. $V_1=\frac{q}{\sqrt{(r\cos\theta-\frac{L}{2})^2+(r\sin\theta-\frac{L}{2})^2}}=\frac{q}{\sqrt{r^2+\frac{L^2}{4}-rL(\cos\theta+\sin\theta)}}$

$r_2=\sqrt{(r\cos\theta-\frac{L}{2})^2+(r\sin\theta+\frac{L}{2})^2}=\sqrt{r^2+\frac{L^2}{4}-rL(\cos\theta-\sin\theta)}$

$r_1=\sqrt{(r\cos\theta+\frac{L}{2})^2+(r\sin\theta-\frac{L}{2})^2}=\sqrt{r^2+\frac{L^2}{4}-rL(\sin\theta-\cos\theta)}$

$r_2=\sqrt{(r\cos\theta+\frac{L}{2})^2+(r\sin\theta+\frac{L}{2})^2}=\sqrt{r^2+\frac{L^2}{4}+rL(\cos\theta+\sin\theta)}$

$\cos\theta=\frac{r}{\sqrt{r^2+\frac{L^2}{4}}}$, $\sin\theta=\frac{\frac{L}{2}}{\sqrt{r^2+\frac{L^2}{4}}}$

19. $\rho(r)=-\frac{q}{\pi a^3}e^{-\frac{2r}{a}}$

$Q=\int_0^{\infty}\int_0^{\pi}\int_0^{2\pi}\rho\,dr\,r\,d\theta\,d\varphi$

$=2\pi\times2\times\int_0^{\infty}-\frac{q}{\pi a^3}e^{-\frac{2r}{a}}r^2\,dr$

$=-\frac{4q}{a^3}\int_0^{\infty}e^{-\frac{2r}{a}}r^2dr$

$=\frac{4q}{a^3}\int_0^{\infty}e^{-\frac{2r}{a}}r^2\,dr$

$\int e^{kr}r\,dr$

$=\frac{1}{k}\int r^2de^{kr}=\frac{1}{k}r^2e^{kr}-\frac{2}{k}\int re^{kr}dr$

$=\frac{1}{k}r^2e^{kr}-\frac{2}{k^2}re^{kr}+\frac{2}{k^2}\int e^{kr}dr$

$=\frac{r^2}{k}e^{kr}-\frac{2r}{k^2}e^{kr}+\frac{2}{k^3}e^{kr}$

$=-\frac{4q}{a^3}(-\frac{a}{2}r^2e^{-\frac{2r}{a}}+\frac{a^2}{4}re^{-\frac{2r}{a}}-\frac{a^3}{8}e^{-\frac{2r}{a}})$

$=\frac{2q}{a^3}re^{-\frac{2r}{a}}-\frac{qr}{2a^2}e^{-\frac{2r}{a}}+\frac{q}{4}e^{-\frac{2r}{a}}$

$=e^{-\frac{2r}{a}}q(\frac{2r^2}{a^3}-\frac{r}{2a^2}+\frac{1}{4})$

$E=4\pi r^2\cdot\frac{1}{\varepsilon_0}\int_0^r e^{-\frac{2r}{a}}q(\frac{2r^2}{a^3}-\frac{r}{2a^2}+\frac{1}{4})\,dr$

$=\frac{8\pi}{\varepsilon_0}\int_0^r e^{-\frac{2r}{a}}\frac{2r^2}{a^3}-e^{-\frac{2r}{a}}\frac{r}{2a^2}+\phi^{-\frac{2r}{a}}\frac{1}{4}\,dr$

$=\frac{8\pi}{\varepsilon_0}\int_0^r\frac{2}{a^3}r^2(-\frac{a}{2}e^{-\frac{2r}{a}})+\frac{1}{a^2}r(\frac{a^2}{4}e^{-\frac{2r}{a}})-\frac{1}{4}\cdot\frac{a^3}{8}e^{-\frac{2r}{a}}\Big|_0^r$

$=(-\frac{r^2}{a}-\frac{r}{2}+\frac{a}{8})e^{-\frac{2r}{a}}$

$-(0-0+\frac{a}{8})$

$=\frac{q}{\varepsilon_0}(1-(\frac{r^2}{a}+\frac{r}{2}+\frac{a}{8})e^{-\frac{2r}{a}})$

$=\frac{q}{\varepsilon_0}(1-\frac{1}{a^2}(r^2+\frac{a}{2}r+\frac{a^2}{8})e^{-\frac{2r}{a}})$

$=\frac{q}{\varepsilon_0}(1-\frac{r^2}{a^2}-\frac{r}{2a}-\frac{1}{8})e^{-\frac{2r}{a}}$, $E=\cdots$

20. $U=\int_r^{\infty}\frac{q}{4\pi\varepsilon_0 r^2}\left[\frac{r^2}{a^2}+\frac{2r}{a}+\frac{1}{2}\right]e^{-\frac{2r}{a}}dr$

$=\frac{q}{4\pi\varepsilon_0}\int_r^{\infty}\left[\left(\frac{r}{a}\right)^2+\frac{2r}{a}+\frac{1}{2}\right]e^{-\frac{2r}{a}}dr$

$=\frac{q}{4\pi\varepsilon_0}\left[\left(\frac{r}{a}\right)^2+\frac{1}{2}\right]e^{-\frac{2r}{a}}$

$\frac{d}{dr}(\frac{r}{a})^2=\frac{2r}{a^2}$, $\frac{d}{dr}e^{-\frac{2r}{a}}=-\frac{2}{a}e^{-\frac{2r}{a}}$

$\delta=\Delta x$ 积分区间 $\Sigma=\sum$ 的极限

$\vec{E}=\frac{\lambda_0\cos\theta\,Rd\theta}{4\pi\varepsilon_0 R^2}\cdot\vec{r}=\vec{E}_0$

$ds=Rd\theta\cdot R$

$\lambda=\lambda_0\cos\theta$

S_1 $\vec{E}=\frac{2k}{2k}\int_0^{\frac{\pi}{2}}\frac{\lambda_0\cos\theta\,Rd\theta}{R^2}\vec{r}_0$

S_2 $dE_x=\frac{\lambda_0\cos\theta\,d\theta}{4\pi\varepsilon_0 R}\cos\theta$

$\alpha=\theta$ $\frac{dx}{4\pi\varepsilon_0 R}=\frac{\lambda_0\cos^2\theta\,d\theta}{4\pi\varepsilon_0 R}$

$E_x=\frac{\lambda_0}{4\pi\varepsilon_0 R}\int_0^{\frac{\pi}{2}}\frac{1+\cos2\theta}{2}d\theta=\frac{\lambda_0}{8\varepsilon_0}\pi$

S_3 $\frac{dE_y}{4\pi\varepsilon_0 R}=\frac{\lambda_0}{8\varepsilon_0}\sin2\theta$

$E_y=\frac{\lambda_0}{8\varepsilon_0}\int_0^{\frac{\pi}{2}}\sin2\theta\,d\theta=\frac{\lambda_0}{8\varepsilon_0}$

$\vec{E}=\frac{\lambda_0}{8\varepsilon_0}\vec{i}+\frac{\lambda_0}{8\varepsilon_0}\vec{j}$

$\delta\to0$ 积分 $\Sigma\to\int$ 的极限

$ds=R\,d\theta$ $\lambda=\lambda_0\cos\theta$

$\vec{E}=\nabla U$, $\left[\frac{r}{(r^2+r^2)^{\frac{3}{2}}}\right]dV$

Gauss

$\vec{E}=\nabla U$